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LIAPUNOV STABILITY ANALYSIS OF AN SCR BRUSHLESS MOTOR DRIVE

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SYMBOLS

B	viscous friction coefficient for this part of motor load.
i_a	instantaneous current in phase a motor stator winding
i_b	instantaneous current in phase b winding
i_c	instantaneous current in phase c winding
i_d	instantaneous direct-axis current
i_q	instantaneous quadrature-axis current
i_R	instantaneous current in rotor winding
i_{do}	steady-state direct-axis current
i_{qo}	steady-state quadrature-axis current
I	crest value of motor line current
$\bar{I}$	motor line current phasor where $\bar{I}$ is related to the instantaneous motor line current by $i = \sqrt{2}$ Real Part of $(\bar{I}\, e^{j\omega t})$
J	total inertia of motor and load
ℓ_{aa}	self inductance of phase a stator winding
ℓ_{bb}	self inductance of phase b winding
ℓ_{cc}	self inductance of phase c winding
$\ell_{ab}=\ell_{ba}$	mutual inductance between phase a and phase b stator windings
$\ell_{bc}=\ell_{cb}$	mutual inductance between phase b and c windings
$\ell_{ac}=\ell_{ca}$	mutual inductance between phase a and c windings
$\ell_{RR}=L_{RR}$	self inductance of rotor winding
$\ell_{aR}=\ell_{Ra}$	mutual inductance between phase a and rotor windings

$\ell_{bR}=\ell_{Rb}$ mutual inductance between phase b and rotor windings

$\ell_{cR}=\ell_{Rc}$ mutual inductance between phase c and rotor windings

L_o leakage inductance of stator winding representing flux linkages/ampere in a stator winding that do not link rotor winding

L_{go} constant part of stator winding self inductance ignoring leakage inductance

L_{g2} maximum value of part of stator winding self inductance which varies with rotor position

$\ell_{RR}=L_{RR}$ self inductance of rotor winding

L_{SR} maximum value of stator-rotor mutual inductance

P number of poles

T fixed torque load on motor

T_D torque developed by motor

v_a instantaneous voltage applied to phase a motor stator winding

v_b instantaneous voltage applied to phase b winding

v_c instantaneous voltage applied to phase c winding

v_d instantaneous direct-axis voltage

v_q instantaneous quadrature-axis voltage

v_R instantaneous voltage applied to rotor winding

V crest value of line to neutral voltage applied to stator windings

δ electrical torque angle

θ electrical angle measured from phase a axis to rotor direct-axis

$\frac{d\theta}{dt}$ electrical angular velocity of rotor

$\frac{d^2\theta}{dt^2}$	electrical angular acceleration of rotor
λ_a	flux linkages of phase a stator winding
λ_b	flux linkages of phase b winding
λ_c	flux linkages of phase c winding
λ_R	flux linkages of rotor winding
φ	power factor angle
ω_o	frequency of voltage applied to stator windings

I. INTRODUCTION

The silicon controlled rectifier (SCR) has generated renewed interest in many static electric power control and conversion techniques. Since the SCR has similar characteristics to a gas thyratron, while possessing all of the advantages of a solid state device, it promised to have considerable impact on the electric power control and conversion technology. This impact has been felt in a wide variety of areas in the eight years since the first announcement by General Electric of the availability of the SCR. A number of concerns now manufacture a range of devices and a large number of products utilizing the SCR are commercially available including static exciters for electrical machines, regulated d-c and a-c power supplies, d-c motor drives, and fixed frequency inverters.

One of the more recent and possibly most promising applications is SCR equipment for variable speed a-c motor drives. This application involves an SCR circuit to provide variable frequency power usually to either a synchronous or induction motor. Such a system provides a variable speed motor control without requiring a commutator type machine. For precision speed control systems, the variable frequency supplied synchronous motor system is preferable. This system is used in equipment manufactured by several concerns for synthetic fibre spinning processes. It is also of considerable interest in defense applications for shaft positioning or speed control as it permits precision control with completely brushless motors thereby providing highly reliable systems.

It is essential for the variable speed element in any speed control system to be stable itself over its required range of operation. This is generally taken for granted since the instability problem studied is usually that caused by networks and control loops external to the electro-mechanical actuator. However, the synchronous motor supplied from a variable frequency power source can be unstable by itself for certain operating conditions. This "hunting" problem of a synchronous motor operated over a wide frequency or speed range is the subject of this dissertation. The experimental model constructed includes a reluctance synchronous motor supplied from an SCR variable frequency cycloconverter circuit, as this is a completely brushless precision speed control system that is appropriate for certain practical applications. One of the simpler theorems of Liapunov is employed in the stability analysis to predict the conditions required for stability and to show the effect on stability when system parameters are varied.

II. BACKGROUND

A. Reluctance Synchronous Motor

The motor analysis in this work parallels that presented in Fitzgerald and Kingsley (1). The differential equations for the motor are written from the coupled circuit viewpoint, and then transformed using the d-q transformation, which is a change of variables that simplifies considerably the differential equations of the motor. These simplified equations are used in the Liapunov stability analysis.

The d-q transformation has its origin from work of Blondel (2, 3), and thus this general method of analyzing a synchronous machine is often referred to as the "Blondel two-reaction method". Extensive background literature has existed for many years relevant to the development of this method and related synchronous machine analytical techniques. Much of the early development was carried out by Doherty and Nickle (4) and somewhat later by Park (5). Since 1950 there have been numerous additional treatments of the analysis of synchronous machines (6-13).

The background literature relative to the reluctance synchronous motor is much more limited (14-17). In the past this motor has not had widespread commercial application because of its relatively low power factor and output torque. However, in recent years machines have been manufactured with larger direct to quadrature axis reactance ratios and accordingly greater torque output. In addition, the availability of solid state variable frequency power supplies has made practical and economic control schemes possible. For these reasons, the reluctance motor has been used in an increasing number of applications where precision speed control of multiple motors is required, and it shows promise for more widespread application in various control systems as it is a simple and reliable machine.

The specific problem which is the major subject of this dissertation--hunting of a reluctance synchronous motor when operated over a wide frequency range--has not been reported in the literature. Concordia (18) has analyzed the somewhat related problem of hunting of a salient-pole synchronous machine during starting. In general, the literature on the transient performance of synchronous machines has been concerned with the response of the machine to load disturbances and different types of faults in the polyphase supply voltage. It was not until practical solid state power supplies became available for delivering variable frequency power to synchronous motors that the hunting problem of such motors when operated at reduced frequency became of definite concern.

B. Liapunov's Direct Method

The fundamental work of Liapunov was first published in Russia in 1892, translated into French in 1907, and reprinted in English in 1949. It has been given serious study in this country only during the past ten years, but it is now considered the most general method of studying the stability of nonlinear systems. The essense of the method is that, without solving the differential equations representing a system, it is possible to reach conclusions regarding stability. In general, the results are sufficient but not necessary conditions for stability or instability.

The main points of Liapunov's theory are described comprehensively in Hahn (19), and the application of the method is treated in a clear and concise fashion in La Salle and Lefschetz (20). A number of additional excellent books and articles have been published regarding the theory and application of the method (21-30).[1]

In the application of the method, the most difficult task is the construction of a Liapunov function. Such a function is required in the hypotheses of most of the important theorems from which conclusions regarding stability or instability can be reached. There is no general method

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[1]Refer to Hahn (19, pp. 151-173) for an extensive bibliography on the subject.

for finding a Liapunov function although several techniques have been and are being studied in this connection (31-34).

Fortunately there are some theorems available which permit one to reach conclusions regarding stability without having to find a specific Liapunov function. The idea behind these is that when the hypotheses of such a theorem are met, it is known that a Liapunov function exists but its explicit form is not required. Since the existence of a certain kind of Liapunov function is assured, conclusions regarding stability can be reached using the results of the fundamental theorems of Liapunov. The differential equations representing the reluctance synchronous motor fortuitously meet the requirements of one of these theorems. This theorem is discussed in Pontryagin (35, theorem 19, pp. 201-211), and it is the key to the stability analysis presented in this dissertation. With Theorem 19 (35) it is possible to determine the conditions for stability of the reluctance synchronous motor operating over a wide frequency range.

No published articles have been found discussing the use of Liapunov's direct method in synchronous machine stability problems.[1] However, Gless (36) has used this method in power system stability analysis.

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[1]The application of Liapunov's direct method to synchronous machine stability problems was suggested to the author in December, 1963, by Dr. George Seifert, Ames, Iowa.

C. SCR Motor Drives

There has long been a desire to eliminate the commutator of a d-c machine, since the brushes and commutator account for the principal maintenance required for trouble free operation of such machines. In 1934 Alexanderson and Mittag (37) described a "Thyratron" motor. This novel development of that era employed the relatively new grid controlled gas rectifiers to accomplish electronically the commutator functions required in a d-c machine. With this approach a machine similar in construction to a synchronous motor was used, the grid controlled rectifiers operated from 4000 volts, 3 phase, 60 cps to deliver controlled power to the motor windings, and the rectifiers were controlled by signals from a "distributor" mechanically coupled to the motor shaft. This approach provides a commutatorless motor with the speed control characteristics of a d-c machine. However, these systems did not receive wide spread application because of their complexity and associated high cost compared to conventional machines.

With the advent of power transistors and SCR's there has been a considerable rebirth of interest in commutatorless d-c motors and all forms of a-c motor speed control. Much of the work to date has involved transistor controls and relatively small motors (38-45). However, there is increasing effort on the development of SCR a-c motor drives for integral horsepower machines (46-52), as these provide high performance and extremely reliable motor speed controls.

III. ANALYSIS

A. Motor

The theory of the ideal synchronous machine as developed by Fitzgerald and Kingsley (1, chapter 5) forms the basis for this development of the differential equations representing the reluctance synchronous motor. In this approach, the machine is regarded as a group of coupled windings with time varying self and mutual inductances. The important assumptions are as follows:

1. A three-phase reluctance synchronous motor with three distributed windings a, b, and c on the stator and one rotor winding
2. Balanced three-phase sinusoidal voltages applied to the stator windings
3. Negligible saturation and iron loss
4. Negligible space harmonics in machine flux
5. A load on the motor shaft including inertia J, viscous friction B, and constant torque T
6. Self and mutual inductances for each of the windings as given below:[1]

$$\ell_{aa} = L_o + L_{go} + L_{g2} \cos 2\theta \tag{1}$$

$$\ell_{bb} = L_o + L_{go} + L_{g2} \cos (2\theta + 120^\circ) \tag{2}$$

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[1] An explanation of the form of these inductances is given in Fitzgerald and Kingsley (1, pp. 225-228).

$$\ell_{cc} = L_o + L_{go} + L_{g2} \cos (2\theta - 120^o) \tag{3}$$

$$\ell_{ab} = \ell_{ba} = -0.5 L_{go} + L_{g2} \cos (2\theta - 120^o) \tag{4}$$

$$\ell_{bc} = \ell_{cb} = -0.5 L_{go} + L_{g2} \cos 2\theta \tag{5}$$

$$\ell_{ac} = \ell_{ca} = -0.5 L_{go} + L_{g2} \cos (2\theta + 120^o) \tag{6}$$

$$\ell_{RR} = L_{RR} \tag{7}$$

$$\ell_{aR} = \ell_{Ra} = L_{SR} \cos \theta \tag{8}$$

$$\ell_{bR} = \ell_{Rb} = L_{SR} \cos (\theta - 120^o) \tag{9}$$

$$\ell_{cR} = \ell_{Rc} = L_{SR} \cos (\theta + 120^o) \tag{10}$$

A script ℓ is used for inductances which vary with shaft position and a capital L denotes a fixed quantity.

The flux linkage relations, the equations arising from applying Kirchhoff's law to each closed circuit, and the expression equating the torque developed by the motor to the load torque, provide a set of equations completely describing the motor. These equations are written below:

$$\lambda_a = \ell_{aa} i_a + \ell_{ab} i_b + \ell_{ac} i_c + \ell_{aR} i_R \tag{11}$$

$$\lambda_b = \ell_{ba} i_a + \ell_{bb} i_b + \ell_{bc} i_c + \ell_{bR} i_R \tag{12}$$

$$\lambda_c = \ell_{ca} i_a + \ell_{cb} i_b + \ell_{cc} i_c + \ell_{cR} i_R \tag{13}$$

$$\lambda_R = \ell_{Ra} i_a + \ell_{Rb} i_b + \ell_{Rc} i_c + \ell_{RR} i_R \tag{14}$$

$$v_a = r_a i_a + \frac{d\lambda_a}{dt} \tag{15}$$

$$v_b = r_b i_b + \frac{d\lambda_b}{dt} \tag{16}$$

$$v_c = r_c\, i_c + \frac{d\lambda_c}{dt} \tag{17}$$

$$v_R = r_R\, i_R + \frac{d\lambda_R}{dt} \tag{18}$$

$$T_D = \frac{2}{P}\, J\, \frac{d^2\theta}{dt^2} + \frac{2}{P}\, B\, \frac{d\theta}{dt} + \frac{2}{P}\, T \tag{19}$$

Since $\frac{d\theta}{dt}$ is used for the electrical angular speed, the constant including the number of poles is required for the first two terms on the right side of Equation 19. The fixed torque load is multiplied by the same constant to simplify subsequent relations. From this point on, matrix notation will be used as much as practical as it provides a shorter method of writing complex sets of equations.

Combining 1-14, the flux linkage relations may be written as in Equation 20. This matrix equation is quite formidable. Thus, it would be most desirable to transform it into a simpler form. This is exactly what is accomplished by a certain transformation called the d-q transformation, originated by Blondel (2) and developed by Doherty and Nickle (4) and Park (5).

Although it may be considered just a change of variables, the d-q transformation can also be given physical significance. The direct-axis is the minimum reluctance axis of the rotor and the quadrature-axis is 90 electrical degrees ahead of the direct-axis, in the direction of rotor rotation. These axes of course rotate with the rotor and so are stationary to an observer on the rotor. This means that if the coordinate

$$
\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_R \end{bmatrix}
=
\begin{bmatrix}
L_o+L_{go} + L_{g2}\cos 2\theta & -0.5L_{go} + L_{g2}\cos(2\theta-120^\circ) & -0.5L_{go} + L_{g2}\cos(2\theta-120^\circ) & L_{SR}\cos\theta \\
-0.5L_{go} + L_{g2}\cos(2\theta-120^\circ) & L_o+L_{go} + L_{g2}\cos(2\theta+120^\circ) & -0.5L_{go} + L_{g2}\cos 2\theta & L_{SR}\cos(\theta-120^\circ) \\
-0.5L_{go} + L_{g2}\cos(2\theta+120^\circ) & -0.5L_{go} + L_{g2}\cos 2\theta & L_o+L_{go} + L_{g2}\cos(2\theta-120^\circ) & L_{SR}\cos(\theta+120^\circ) \\
L_{SR}\cos\theta & L_{SR}\cos(\theta-120^\circ) & L_{SR}\cos(\theta+120^\circ) & L_{RR}
\end{bmatrix}
\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_R \end{bmatrix}
\qquad (20)
$$

Matrix Plate 1. Equation 20

references for the system are the d-q axes, since the reluctances along these axes are fixed, the associated flux linkages and inductances will not vary with shaft rotation. This should therefore result in a considerable simplification of the flux linkage relations. That this is indeed the case will be shown by the application of the d-q transformation to 20.

The d-q transformation may be defined in the following manner:

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = A \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{21}$$

where

$$A = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-120^\circ) & \cos(\theta+120^\circ) \\ -\sin\theta & -\sin(\theta-120^\circ) & -\sin(\theta+120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{22}$$

The currents i_d, i_q, and i_o are the new variables to which the phase currents i_a, i_b, and i_c are transformed by the transformation matrix A. The same transformation is used to relate λ_d, λ_q, λ_o to λ_a, λ_b, and λ_c and v_d, v_q, v_o to v_a, v_b, and v_c. The inverse transformation is abtained by premultiplying both sides of 21 by the matrix A^{-1}, the inverse of A. This gives

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = A^{-1} \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} \tag{23}$$

and the inverse of A may be obtained in a conventional manner yielding

$$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-120^\circ) & -\sin(\theta-120^\circ) & 1 \\ \cos(\theta+120^\circ) & -\sin(\theta+120^\circ) & 1 \end{bmatrix} \tag{24}$$

Equation 20 is transformed most easily by rewriting it first as the two separate matrix Equations 25 and 26. To illustrate the procedure from here on, for example using 23 and 24 in 26 gives 27. Equation 27 may be simplified using trigonometric identities as follows:

$$\begin{aligned} \cos^2\theta+\cos^2(\theta-120^\circ)+\cos^2(\theta+120^\circ) &= \tfrac{1}{2}+\tfrac{1}{2}\cos 2\theta+\tfrac{1}{2}+\tfrac{1}{2}\cos2(\theta-120^\circ) \\ &\quad +\tfrac{1}{2}+\tfrac{1}{2}\cos2(\theta+120^\circ) \\ &= \frac{3}{2}+\tfrac{1}{2}[\cos2\theta+\cos(2\theta+120^\circ) \\ &\quad +\cos(2\theta-120^\circ)] \\ &= \frac{3}{2}+\tfrac{1}{2}[\cos2\theta+2(\cos2\theta) \\ &\quad (\cos 120^\circ)] \\ &= \frac{3}{2}+\tfrac{1}{2}[\cos2\theta-\cos2\theta] = \frac{3}{2} \end{aligned} \tag{28}$$

$$\begin{aligned} \sin\theta\cos\theta+\sin(\theta-120^\circ)\cos(\theta-120^\circ) &= \tfrac{1}{2}\sin2\theta+\tfrac{1}{2}\sin2(\theta-120^\circ) \\ +\sin(\theta+120^\circ)\cos(\theta+120^\circ) &\quad +\tfrac{1}{2}\sin2(\theta+120^\circ) \\ &= \tfrac{1}{2}[\sin2\theta+\sin(2\theta+120^\circ) \\ &\quad +\sin(2\theta-120^\circ)] \end{aligned}$$

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \begin{bmatrix} L_o + L_{go} + L_{g2}\cos 2\theta & -0.5L_{go} + L_{g2}\cos(2\theta - 120^\circ) & -0.5L_{go} + L_{g2}\cos(2\theta + 120^\circ) \\ -0.5L_{go} + L_{g2}\cos(2\theta - 120^\circ) & L_o + L_{go} + L_{g2}\cos(2\theta + 120^\circ) & -0.5L_{go} + L_{g2}\cos 2\theta \\ -0.5L_{go} + L_{g2}\cos(2\theta + 120^\circ) & -0.5L_{go} + L_{g2}\cos 2\theta & L_o + L_{go} + L_{g2}\cos(2\theta - 120^\circ) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$+ \begin{bmatrix} L_{SR} \cos \theta \\ L_{SR} \cos (\theta - 120^\circ) \\ L_{SR} \cos (\theta + 120^\circ) \end{bmatrix} i_R$$

$$= B \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + C\, i_R \qquad (25)$$

Matrix Plate 2. Equation 25

$$\lambda_R = [L_{SR}\cos\theta \quad L_{SR}\cos(\theta-120^\circ) \quad L_{SR}\cos(\theta+120^\circ)] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + L_{RR}\, i_R \tag{26}$$

$$\lambda_R = [L_{SR}\cos\theta \quad L_{SR}\cos(\theta-120^\circ) \quad L_{SR}\cos(\theta+120^\circ)] \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-120^\circ) & -\sin(\theta-120^\circ) & 1 \\ \cos(\theta+120^\circ) & -\sin(\theta+120^\circ) & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix}$$

$$+ L_{RR}\, i_R$$

$$= L_{SR} \begin{bmatrix} \cos^2\theta + \cos^2(\theta-120^\circ) + \cos^2(\theta+120^\circ) \\ -\sin\theta\cos\theta - \sin(\theta-120^\circ)\cos(\theta-120^\circ) - \sin(\theta+120^\circ)\cos(\theta+120^\circ) \\ \cos\theta + \cos(\theta-120^\circ) + \cos(\theta+120^\circ) \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix}$$

$$+ L_{RR}\, i_R \tag{27}$$

Matrix Plate 3. Equations 26 and 27

$$= \frac{1}{2}\,[\sin 2\theta + 2\sin 2\theta\,[\cos 120^{o}]$$

$$= \frac{1}{2}\,[\sin 2\theta - \sin 2\theta] = 0 \qquad (29)$$

$$\cos\theta + \cos(\theta - 120^{o}) + \cos(\theta + 120^{o}) = \cos\theta + 2\cos\theta\cos(120^{o}) = 0 \qquad (30)$$

Thus, 27 is reduced to

$$\lambda_R = \frac{3}{2} L_{SR}\, i_d + L_{RR}\, i_R \qquad (31)$$

Equation 25 is transformed similarly by rewriting it in terms of d-q quantities

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix} = A \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = A\ B \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + A\ C\ i_R$$

$$= A\ B\ A^{-1} \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} + A\ C\ i_R \qquad (32)$$

Although it is quite laborious, it is perfectly straightforward to perform the matrix multiplications indicated in 32. After using trigonometric identities similar to those used in the simplification of 27, the following results

$$\lambda_d = L_{SR}\, i_R + L_d\, i_d \qquad (33)$$

$$\lambda_q = L_q i_q \qquad (34)$$

$$\lambda_o = L_o\, i_o \qquad (35)$$

where

$$L_d = L_o + \frac{3}{2}\,(L_{go} + L_{g2}) \qquad (36)$$

and

$$L_q = L_o + \frac{3}{2}(L_{go} - L_{g2}) \tag{37}$$

Therefore, the formidable Equation 20 containing non-constant inductances has been simplified immensely to 31, 33, 34, and 35 which contain only fixed inductance values.

By a similar process, Equations 15-17 may be transformed to the d-q domain. In the following it is assumed that the resistances of the stator windings are the same, i.e.,

$$r_a = r_b = r_c = r_S \tag{38}$$

Equations 15-17 may be written

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = r_S \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} \tag{39}$$

and using 21 and 23 for the voltages, currents, and flux linkages gives

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} = A \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = A\, r_S \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + A \frac{d}{dt}\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix}$$

$$= r_S\, AA^{-1} \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} + A \frac{d}{dt} A^{-1} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix}$$

$$= r_S \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} + A\dot{A}^{-1} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix} \tag{40}$$

where

$$\dot{A}^{-1} = \frac{d}{dt} [A^{-1}] \tag{41}$$

To evaluate the second term on the right side of 40, first consider Equation 42. Again using trigonometric identities, this can be reduced to 43. Whenever the forms $\sin \gamma + \sin(\gamma+120^\circ) + \sin(\gamma-120^\circ)$ or $\cos \gamma + \cos(\gamma+120^\circ) + \cos(\gamma-120^\circ)$ appear, it is known that these must be zero since they can be considered the sum of balanced unit amplitude 3-phase voltages, which are known to be zero. Equation 40 may then be written

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} = r_S \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} + \begin{bmatrix} 0 & -\frac{d\theta}{dt} & 0 \\ \frac{d\theta}{dt} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix} \tag{44}$$

or

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} = r_S \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} + \begin{bmatrix} -\lambda_q \frac{d\theta}{dt} \\ \lambda_d \frac{d\theta}{dt} \\ 0 \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix} \tag{45}$$

As shown in Fitzgerald and Kingsley (1, p. 233), the

$$A\dot{A}^{-1} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-120^\circ) & \cos(\theta+120^\circ) \\ -\sin\theta & -\sin(\theta-120^\circ) & -\sin(\theta+120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} -\sin\theta & -\cos\theta & 0 \\ -\sin(\theta-120^\circ) & -\cos(\theta-120^\circ) & 0 \\ -\sin(\theta+120^\circ) & -\cos(\theta+120^\circ) & 0 \end{bmatrix}\frac{d\theta}{dt}$$

$$= \frac{2}{3}\frac{d\theta}{dt}\begin{bmatrix} \begin{matrix} -\sin\theta\cos\theta \\ -\sin(\theta-120^\circ)\cos(\theta-120^\circ) \\ -\sin(\theta+120^\circ)\cos(\theta+120^\circ) \end{matrix} & \begin{matrix} -\cos^2\theta \\ -\cos^2(\theta-120^\circ) \\ -\cos^2(\theta+120^\circ) \end{matrix} & 0 \\ \begin{matrix} \sin^2\theta \\ +\sin^2(\theta-120^\circ) \\ +\sin^2(\theta+120^\circ) \end{matrix} & \begin{matrix} \sin\theta\cos\theta \\ +\sin(\theta-120^\circ)\cos(\theta-120^\circ) \\ +\sin(\theta+120^\circ)\cos(\theta+120^\circ) \end{matrix} & 0 \\ \begin{matrix} -\frac{1}{2}\sin\theta \\ -\frac{1}{2}\sin(\theta-120^\circ) \\ -\frac{1}{2}\sin(\theta+120^\circ) \end{matrix} & \begin{matrix} -\frac{1}{2}\cos\theta \\ -\frac{1}{2}\cos(\theta-120^\circ) \\ -\frac{1}{2}\cos(\theta+120^\circ) \end{matrix} & 0 \end{bmatrix} \qquad (42)$$

Matrix Plate 4. Equation 42

$$
A\dot{A}^{-1} = \frac{2}{3}\frac{d\theta}{dt}\begin{bmatrix}
\begin{matrix}-\frac{1}{2}\sin 2\theta \\ +\sin(2\theta+120^\circ) \\ +\sin(2\theta-120^\circ)\end{matrix} & \begin{matrix}-\frac{1}{2}-\frac{1}{2}\cos 2\theta \\ -\frac{1}{2}-\frac{1}{2}\cos(2\theta+120^\circ) \\ -\frac{1}{2}-\frac{1}{2}\cos(2\theta-120^\circ)\end{matrix} & 0 \\
\begin{matrix}\frac{1}{2}-\frac{1}{2}\cos 2\theta \\ +\frac{1}{2}-\frac{1}{2}\cos(2\theta+120^\circ) \\ +\frac{1}{2}-\frac{1}{2}\cos(2\theta-120^\circ)\end{matrix} & \begin{matrix}\frac{1}{2}\sin 2\theta \\ +\sin(2\theta+120^\circ) \\ +\sin(2\theta-120^\circ)\end{matrix} & 0 \\
-\frac{1}{2}(\sin\theta+2\sin\theta\cos 120^\circ) & -\frac{1}{2}(\cos\theta+2\cos\theta\cos 120^\circ) & 0
\end{bmatrix}
$$

$$
= \frac{2}{3}\frac{d\theta}{dt}\begin{bmatrix} 0 & -3/2 & 0 \\ 3/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (43)
$$

Matrix Plate 5. Equation 43

torque developed by the synchronous machine is given by

$$T_D = \frac{3}{2}\frac{P}{2}(\lambda_d i_q - \lambda_q i_d) \qquad (46)$$

Combining Equations 18 and 31, substituting 33 and 34 into 45, and combining 19, 33, 34 and 46 shows the final result of the d-q transformation to be the following

$$v_R = r_R i_R + L_{RR}\frac{di_R}{dt} + \frac{3}{2}L_{SR}\frac{di_d}{dt} \qquad (47)$$

$$v_d = r_S i_d + L_d\frac{di_d}{dt} + L_{SR}\frac{di_R}{dt} - L_q i_q \frac{d\theta}{dt} \qquad (48)$$

$$v_q = r_S i_q + L_q\frac{di_q}{dt} + L_{SR} i_R \frac{d\theta}{dt} + L_d i_d \frac{d\theta}{dt} \qquad (49)$$

$$\frac{3}{8}P^2(L_{SR} i_R i_q + L_d i_d i_q - L_q i_d i_q) = J\frac{d^2\theta}{dt^2} + \frac{Bd\theta}{dt} + T \qquad (50)$$

The particular motor under study is a reluctance synchronous machine with a squirrel cage rotor winding. This is considered an equivalent single winding on the rotor for this work. As there is no rotor applied voltage

$$v_R = 0 \qquad (51)$$

The applied voltages to the stator windings are

$$v_a = V\cos\omega_o t \qquad (52)$$

$$v_b = V\cos(\omega_o t - 120^o) \qquad (53)$$

$$v_c = V\cos(\omega_o t + 120^o) \qquad (54)$$

The voltages may be transformed to d-q variables using the voltage relations similar to 21 and 22, giving

$$v_d = V \cos (\omega_o t - \theta) \tag{55}$$

$$v_q = V \sin (\omega_o t - \theta) \tag{56}$$

In subsequent work, the v_o, i_o, and λ_o components are ignored as they are zero for balanced three phase systems. Since the machine to be used in the experimental results is a 4 pole motor, P = 4 will also be used to simplify the relations so that 47 through 50 become

$$r_R\, i_R + L_{RR} \frac{di_R}{dt} + \frac{3}{2} L_{SR} \frac{di_d}{dt} = 0 \tag{57}$$

$$r_S\, i_d + L_d \frac{di_d}{dt} + L_{SR} \frac{di_R}{dt} - L_q\, i_q \frac{d\theta}{dt} - V \cos(\omega_o t - \theta) = 0 \tag{58}$$

$$r_S\, i_q + L_q \frac{di_q}{dt} + L_{SR}\, i_R \frac{d\theta}{dt} + L_d i_d \frac{d\theta}{dt} - V \sin(\omega_o t - \theta) = 0 \tag{59}$$

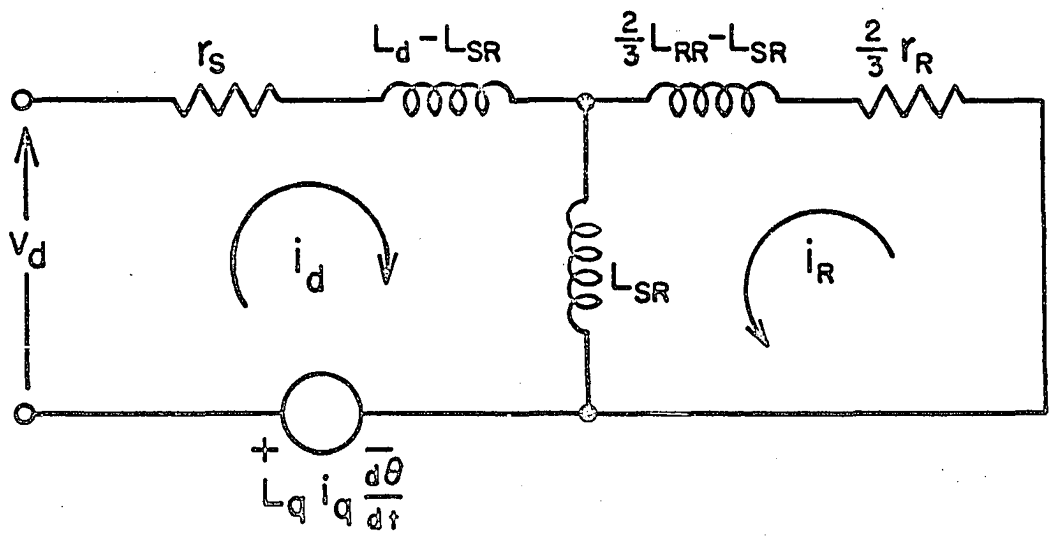
$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} + T_{\sim} - 6\, L_{SR}\, i_R i_q - 6(L_d - L_q) i_d i_q = 0 \tag{60}$$

These are the equations to be used in the Liapunov stability analysis.

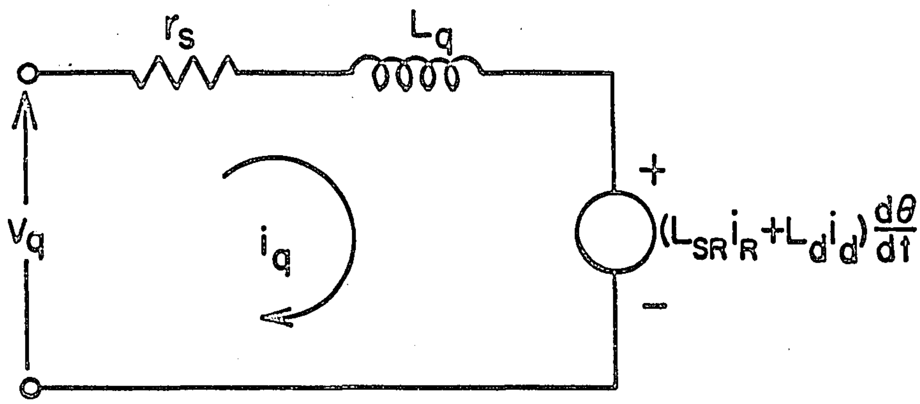
An equivalent circuit of the machine as expressed by 57-59 is given in Fig. 1.

B. Application of Liapunov's Direct Method

There are many different types of stability but for the purpose of this dissertation asymptotic stability is of most interest. In fact this type of stability is generally of most importance to the control engineer. It is defined rigorously in Pontryagin (35, p. 202), Hahn (19, p. 6) or



DIRECT AXIS



QUADRATURE AXIS

FIG. I — EQUIVALENT CIRCUIT OF RELUCTANCE SYNCHRONOUS MOTOR WITH SINGLE CLOSED ROTOR WINDING

La Salle and Lefschetz (20, p. 32). Roughly, a system is asymptotically stable if when it is started sufficiently near an equilibrium or steady-state condition, it asymptotically approaches the equilibrium condition as time increases indefinitely. This implies enough damping so that the system does not continually oscillate in response to a sufficiently small perturbation. It should be noted that this is strictly a dynamic property of a system and is not intended to include such factors as long term drift caused by environmental temperature changes or aging of components. For the remainder of this dissertation the terms asymptotic stability and stability, or asymptotically stable and stable, will be used interchangeably for brevity, with asymptotic stability understood in all cases.

In general, Liapunov's direct method involves finding a certain function called a Liapunov function for the system under study. If this function is positive definite and its time derivative is negative definite, the system is asymptotically stable.[1] The basic idea of the method may be related to the concept of a physical system always seeking a minimum energy equilibrium position. In fact, particularly for simple systems the Liapunov function often can be the total energy.

---

[1]This matter is discussed at much greater length with precise definitions of terms in La Salle and Lefschetz (20, pp. 33-40).

If the derivative of this function is to be negative definite it means that the energy must always be decreasing as the equilibrium position is approached. This is a characteristic of a stable point for a physical system.

The problem is that for most practical cases of any complexity, it is extremely difficult to find a suitable Liapunov function. After some searching, a suitable Liapunov function has not been found for the reluctance synchronous motor. However, most fortunately there are certain theorems which enable one to reach conclusions regarding stability without having to determine explicitly a Liapunov function. For this work it is possible to use such a theorem discussed by Pontryagin (35, Theorem 19). The idea is that if the system under study meets the hypotheses of the theorem, a suitable Liapunov function is known to exist. Therefore, the key to the stability analysis for this work is the application of Theorem 19 of Pontryagin (35) to the equations developed in the previous section.

The essence of Theorem 19 (35) is that _if_ the eigenvalues of a certain matrix, determined from the system equations, all have negative real parts, _then_ the system is asymptotically stable. As a first step in getting the required matrix, it is necessary to write the differential equations representing the motor as an equivalent system of first order differential equations. To this end, multiplying 57 by $\frac{2}{3}\frac{L_d}{L_{SR}}$ and subtracting from 58 eliminates one derivative term to give

$$\frac{di_R}{dt} = -\frac{2}{3}\frac{r_R\,L_d}{L_1\,L_{SR}}\,i_R + \frac{r_S}{L_1}\,i_d$$

$$-\frac{L_q}{L_1}\,i_q\,\frac{d\theta}{dt} - \frac{V}{L_1}\cos\,(\omega_o t-\theta) \qquad (61)$$

where

$$L_1 = \frac{2}{3}\frac{L_d L_{RR}}{L_{SR}} - L_{SR} \qquad (62)$$

Similarly, multiplying 57 by $\frac{L_{SR}}{L_{RR}}$ and subtracting from 58 yields

$$\frac{di_d}{dt} = \frac{r_R\,L_{SR}}{L_2\,L_{RR}}\,i_R - \frac{r_S}{L_2}\,i_d$$

$$+\frac{L_q}{L_2}\,i_q\,\frac{d\theta}{dt} + \frac{V}{L_2}\cos\,(\omega_o t-\theta) \qquad (63)$$

where

$$L_2 = L_d - \frac{3}{2}\frac{{L_{SR}}^2}{L_{RR}} \qquad (64)$$

The following change of variables is now used principally to simplify the notation

$$i_R = x_1;\quad i_d = x_2;\quad i_q = x_3;\quad \theta = x_4;\quad \frac{d\theta}{dt} = x_5 \qquad (65)$$

reducing 59, 60, 61, and 63 to

$$\dot{x}_1 = -\frac{2}{3}\frac{r_R L_d}{L_1 L_{SR}}\,x_1 + \frac{r_S}{L_1}\,x_2$$

$$-\frac{L_q}{L_1}\,x_3 x_5 - \frac{V}{L_1}\cos\,(\omega_o t-x_4) \qquad (66)$$

$$\dot{x}_2 = \frac{r_R L_{SR}}{L_2 L_{RR}} x_1 - \frac{r_S}{L_2} x_2$$

$$+ \frac{L_q}{L_2} x_2 x_5 + \frac{V}{L_2} \cos (\omega_o t - x_4) \tag{67}$$

$$\dot{x}_3 = - \frac{L_{SR}}{L_q} x_1 x_5 - \frac{L_d}{L_q} x_2 x_5$$

$$- \frac{r_S}{L_q} x_3 + \frac{V}{L_q} \sin (\omega_o t - x_4) \tag{68}$$

$$\dot{x}_4 = x_5 \tag{69}$$

$$\dot{x}_5 = 6 \frac{L_{SR}}{J} x_1 x_3 + 6 \frac{(L_d - L_q)}{J} x_2 x_3$$

$$- \frac{B}{J} x_5 - \frac{T}{J} \tag{70}$$

Equations 66-70 can be considered a vector system of differential equations of the form

$$\dot{\vec{x}} = \vec{f}(\vec{x},t) \quad \text{or} \quad \dot{x}_i = f_i(x_j,t); \quad i,j = 1,2,3,4,5 \tag{71}$$

However, the origin is not a critical point for this system as $\vec{f}(\vec{0}) \neq \vec{0}$. Since this is a requirement in Theorem 19 (35), another change of variables is made shifting the origin to a critical point. Letting

$$x_1 = \xi_1; \; x_2 = \xi_2 + i_{do}; \; x_3 = \xi_3 + i_{qo}; \; x_4 = \xi_4 + \omega_o t - 90 - \delta; \; x_5 = \xi_5 + \omega_o \tag{72}$$

and then rewriting 66-70 in terms of these new variables, gives

$$\dot{\xi}_1 = -\frac{2}{3}\frac{r_R L_d}{L_1 L_{SR}}\xi_1 + \frac{r_S}{L_1}(\xi_2 + i_{do})$$

$$-\frac{L_q}{L_1}(\xi_3 + i_{qo})(\xi_5 + \omega_o) - \frac{V}{L_1}\sin(\xi_4 - \delta) \tag{73}$$

$$\dot{\xi}_2 = \frac{r_R L_{SR}}{L_2 L_{RR}}\xi_1 - \frac{r_S}{L_2}(\xi_2 + i_{do})$$

$$+\frac{L_q}{L_2}(\xi_3 + i_{qo})(\xi_5 + \omega_o) + \frac{V}{L_2}\sin(\xi_4 - \delta) \tag{74}$$

$$\dot{\xi}_3 = -\frac{L_{SR}}{L_q}\xi_1(\xi_5 + \omega_o) - \frac{L_d}{L_q}(\xi_2 + i_{do})(\xi_5 + \omega_o)$$

$$-\frac{r_S}{L_q}(\xi_3 + i_{qo}) + \frac{V}{L_q}\cos(\xi_4 - \delta) \tag{75}$$

$$\dot{\xi}_4 = \xi_5 \tag{76}$$

$$\dot{\xi}_5 = 6\frac{L_{SR}}{J}\xi_1(\xi_3 + i_{qo}) + 6\frac{(L_d - L_q)}{J}(\xi_2 + i_{do})(\xi_3 + i_{qo})$$

$$-\frac{B}{J}(\xi_5 + \omega_o) - \frac{T}{J} \tag{77}$$

This is an autonomous vector system of the form

$$\dot{\vec{\xi}} = \vec{f}(\vec{\xi}) \quad \text{or} \quad \dot{\xi}_i = f_i(\xi_j);\ i,j = 1,2,3,4,5 \tag{78}$$

where

$$\vec{f}(\vec{0}) = \vec{0} \tag{79}$$

That the origin is a critical point of the vector system represented by 73-77 may be seen easily by finding relationships from 66-70 for the equilibrium condition, substituting these into 73-77, and observing that the right hand sides of 73-77 will all be zero at $\vec{\xi} = \vec{0}$.

$$\frac{\partial \vec{f}(\vec{\xi})}{\partial \vec{\xi}}(\vec{0}) = \begin{bmatrix} -\frac{2}{3}\frac{r_R L_d}{L_1 L_{SR}} & \frac{r_S}{L_1} & -\frac{\omega_o L_q}{L_1} & -\frac{V\cos\delta}{L_1} & -\frac{L_q i_{qo}}{L_1} \\ \frac{r_R L_{SR}}{L_2 L_{RR}} & -\frac{r_S}{L_2} & \frac{\omega_o L_q}{L_2} & V\frac{\cos\delta}{L_2} & \frac{L_q i_{qo}}{L_2} \\ -\frac{\omega_o L_{SR}}{L_q} & -\frac{\omega_o L_d}{L_q} & -\frac{r_S}{L_q} & V\frac{\sin\delta}{L_q} & -\frac{L_d i_{do}}{L_q} \\ 0 & 0 & 0 & 0 & 1 \\ 6\frac{L_{SR} i_{qo}}{J} & 6\frac{(L_d - L_q) i_{qo}}{J} & 6\frac{(L_d - L_q) i_{do}}{J} & 0 & -\frac{B}{J} \end{bmatrix} \tag{80}$$

Matrix Plate 6. Equation 80

The next step in the application of Theorem 19 (35) is to write the matrix 80. Finally, the essential statement of the theorem is that <u>if</u> the eigenvalues of the above matrix all have negative real parts, <u>then</u> the system is asymptotically stable. Note that this is a sufficient but not necessary condition for asymptotic stability.

It is of course not a simple matter to determine the eigenvalues of a matrix of rank four or more. However, when numbers are available for each of the constants, it is a reasonably simple task to calculate the eigenvalues of the 5 x 5 matrix using a digital computer.[1]

It is also possible to ascertain that the eigenvalues of a matrix all have negative real parts without explicitly determining them. A necessary and sufficient condition for this purpose is discussed in Pontryagin (35, pp. 60-62) and Saaty and Bram (53, p. 196). This is a theorem attributed to Routh-Hurwitz which is a variation of the familiar Routh's criterion for determining whether any of the roots of a polynomial lie in the right half plane. It is necessary first to determine the characteristic polynomial of the matrix from the following determinant:

$$|\lambda I - A| = a_o\lambda^n + a_1\lambda^{n-1} + \ldots + a_n \qquad (81)$$

[1]Dr. R. J. Lambert, Iowa State University, Ames, Iowa, suggested this to the author in July, 1965.

Theorem 7 of Pontryagin (35, p. 60) states that for a 5 x 5 matrix, all of the eigenvalues have negative real parts if and only if all of the following inequalities hold:

$$a_o > 0 \;, \quad a_1 > 0 \tag{82}$$

$$\begin{vmatrix} a_1 & a_3 \\ a_o & a_2 \end{vmatrix} > 0 \tag{83}$$

$$\begin{vmatrix} a_1 & a_3 & a_5 \\ a_o & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} > 0 \tag{84}$$

$$\begin{vmatrix} a_1 & a_3 & a_5 & 0 \\ a_o & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_o & a_2 & a_4 \end{vmatrix} > 0 \tag{85}$$

$$\begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 \\ a_o & a_2 & a_4 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & a_o & a_2 & a_4 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 \end{vmatrix} > 0 \tag{86}$$

Therefore, it is possible to determine necessary and sufficient conditions for all of the eigenvalues of the 5 x 5 matrix to have negative real parts from straightforward calculations using the coefficients of the terms in the characteristic polynomial. This method is used in the next section for the computer calculations. It involves less computer time than is required to calculate the eigenvalues of the 5 x 5 matrix.

In the application of Theorem 7 of Pontryagin (35, p. 60), it is necessary to substitute the numberical values for each of the quantities appearing in the matrix 80 to obtain numbers for each of the elements in this matrix. Thus, the resistances, inductances, frequency, friction coefficient, and inertia must be known. In addition the steady-state or equilibrium conditions must be calculated. These are determined from Equations 74, 75, and 77 evaluated with $\dot{\vec{\xi}} = \vec{0}$ and $\vec{\xi} = \vec{0}$. This gives

$$-r_S\, i_{do} + \omega_o L_q\, i_{qo} = V \sin\delta \tag{87}$$

$$\omega_o L_d\, i_{do} + r_S\, i_{qo} = V \cos\delta \tag{88}$$

$$6(L_d - L_q)\, i_{do}\, i_{qo} = B\omega_o + T \tag{89}$$

Solving 87 and 88 simultaneously for i_{do} and i_{qo} and rewriting 89 gives

$$i_{do} = \frac{V(\omega_o L_q \cos\delta - r_S \sin\delta)}{r_S^2 + \omega_o^2 L_d L_q} \tag{90}$$

$$i_{qo} = \frac{V(\omega_o L_d \sin\delta + r_S \cos\delta)}{r_S^2 + \omega_o^2 L_d L_q} \tag{91}$$

$$i_{do}\, i_{qo} = \frac{B\omega_o + T}{6(L_d - L_q)} \tag{92}$$

It is also possible to combine the above three equations to obtain a fairly complicated quadratic expression in $\sin^2\delta$. This is done in the next section to determine δ for the computer calculations.

A phasor diagram for the machine is given in Fig. 2, using the steady state relations of 87 and 88. The direct-axis current is somewhat arbitrarily chosen along the positive real axis, though this is suggested by the d-q transformation as expressed in 23 and 24. With $i_o = 0$, these equations give

$$i_a = i_d \cos\theta - i_q \sin\theta$$
$$= i_d \cos\theta + i_q \cos(\theta + 90^\circ) \qquad (93)$$

Defining the phasor $\bar{I}_a$ by the following relation

$$i_a = \sqrt{2} \text{ Real Part of } (\bar{I}_a e^{j\theta}) \qquad (94)$$

means that the phasor corresponding to 93 is

$$\bar{I}_a = I_d + j I_q \qquad (95)$$

$$\text{where } I_d = \frac{i_d}{\sqrt{2}} \qquad (96)$$

$$\text{and } I_q = \frac{i_q}{\sqrt{2}} \qquad (97)$$

Therefore, when the above phasor definition is assumed, the real component of $\bar{I}_a$ is I_d and the imaginary component leads this by 90° and is equal to I_q. From 87 and 88

$$-\frac{V}{\sqrt{2}} \sin\delta = r_s I_d - x_q I_q \qquad (98)$$

$$\frac{V}{\sqrt{2}} \cos\delta = r_s I_q + x_d I_d \qquad (99)$$

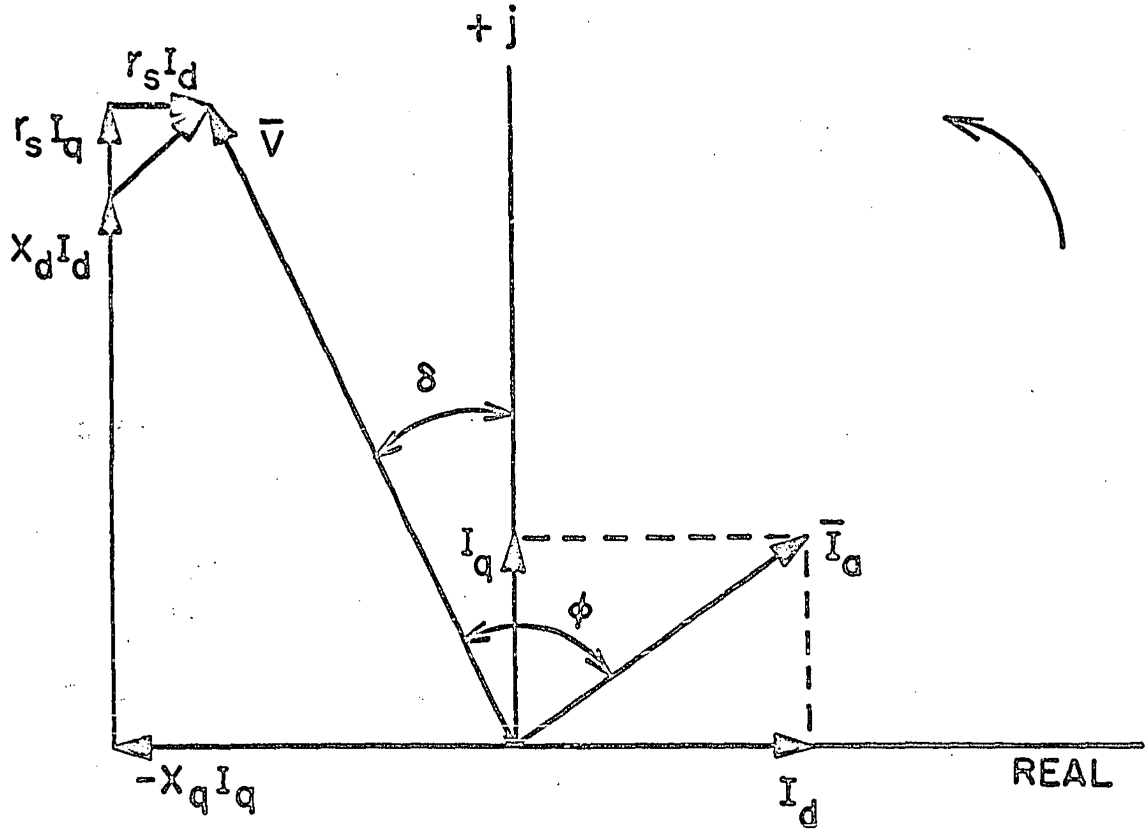


FIG. 2 — PHASOR DIAGRAM OF RELUCTANCE SYNCHRONOUS MOTOR

where

$$x_q = \omega_o L_q \tag{100}$$

and

$$x_d = \omega_o L_d \tag{101}$$

The phasor diagram of Fig. 2 is drawn using Equations 95, 98, and 99. Although such a diagram does not add anything new to the analytical development, it does help to give a better physical understanding of the motor.

C. Computer Calculations

These calculations involve a two-step process. First, the terms of the matrix 80 are calculated using the motor constants and computed values for i_{do}, i_{qo}, and δ from 90, 91, and 92. The motor constants are those determined in Section IV., A., although several different values are used where the particular constant is not known precisely and where it is desired to determine the effect of changing a certain constant. The constant T is assumed zero representing the case of no fixed torque loads. Next, the determinants in 83-86 are evaluated to check for asymptotic stability. It was found for all of the calculations that the conditions of 83 and 84 were always satisfied. However, the determinants in 85 and 86 are positive or negative depending on the particular constants. In fact, 85 and 86 are always found to have the same sign since all of the coefficients of the characteristic equation are positive. As seen from 85 and

86, when a_5 is positive, the determinant in 86 must have the same sign as the determinant of 85. Thus, in the tables that follow the determinant in 85 is tabulated and when it is positive, asympototic stability must occur.

On the tabulated computer results, all of the values of the coefficients a_0-a_5 of the characteristic polynomial in 81 and the determinants in 83-86 are printed out. However, for brevity, only the determinant of 85 is tabulated here as it is sufficient to define the condition for asymptotic stability. The value of this determinant is rounded off, also for brevity, in the tables, while the tabulated computer results include 8 significant figures.

Occasionally, a value for the determinant in 85 is not shown in the tables. In these cases, the computer did not print out a number because of the way the program was set up. It is believed that these values could be determined by a further refinement in the computer program. This is not significant to the performance of the motor but merely the nature of the particular computer set up.

In going from one table to the next, the parameters which are different from those used in the previous table are underlined to make it easier to see the changes which are being made. The voltage range at a given frequency is selected to maintain about the same volts/cycle range at any frequency.

Table 1. $L_d = .105$, $L_q = .035$, $L_{SR} = .03$, $r_R = 1.2$, $J = 2.5x10^{-3}$, $B = .4x10^{-3}$

V	ω	10^{-19} x Determinant in 85 $r_s=1.2$	$r_s=1.7$	$r_s=3.2$
110	377	-37.1	-82.8	-335
100	377	-25.1	-57.6	-240
90	377	-15.9	-38.3	-166
80	377	-	-23.9	-111
70	377	-4.38	-13.5	-69.6
60	377	-1.04	-6.1	-40
50	377	+1.23	-1.03	-19.1
40	377	+2.82	+2.49	-4.54
30	377	+4.24	+5.37	+6.49
90	300	-18.5		
80	300	-11.9		
70	300	-7.16		
60	300	-3.9		
50	300	-		
40	300	-.411		
30	300	+.405		
20	300	+1.01		
70	220	-7.11		
60	220	-4.14		
50	220	-2.2		
40	220	-1.01		
30	220	-.351		
20	220	-.015		
10	220	+1.92		

Table 1 (Continued)

V	ω	10^{-19} x Determinant in 85 r_s=1.2	r_s=1.7	r_s=3.2
45	150	-1.29	-2.67	-9.98
40	150	-.88	-1.83	-6.97
35	150	-.572	-1.2	-4.67
30	150	-.35	-.746	-2.98
25	150	-.198	-.431	-1.79
20	150	-.01	-.224	-.97
15	150	-.039	-.095	-.443
10	150	-.004	-.019	-.12
30	75	-.087		
25	75	-.082		
20	75	-.053		
15	75	-.026		
10	75	-.009		
5	75	-.001		
12	37	$+14.8 \times 10^{-3}$		
11	37	$+7.6 \times 10^{-3}$		
10	37	$+3.2 \times 10^{-3}$		
9	37	$+.78 \times 10^{-3}$		
8	37	$-.42 \times 10^{-3}$		
7	37	$-.86 \times 10^{-3}$		
6	37	$-.89 \times 10^{-3}$		
5	37	$-.73 \times 10^{-3}$		
4	37	$-.5 \times 10^{-3}$		
3	37	$-.28 \times 10^{-3}$		

Table 2. L_d= .105, L_q= .035, L_{SR}= .03, r_R= 0.6, J= $2.5x10^{-3}$, B= $0.5x10^{-3}$

V	ω	10^{-19} x Determinant in 85 r_s=1.2	r_s=1.7	r_s=3.2
110	377	-26.5	-60.6	-245
100	377	-16.7	-39.8	-167
90	377	-9.65	-24.6	-109
80	377	-4.75	-13.8	-67.4
70	377	-1.55	-6.6	-38.3
60	377	-.384	-2.03	-19
50	377	+1.44	+.662	-6.97
40	377	+2	+2.18	+.272
30	377	+2.52	+3.37	+5.34
90	300	-13.9		
80	300	-8.38		
70	300	-4.62		
60	300	-2.2		
50	300	-.766		
40	300	-.011		
30	300	+.338		
20	300	+.574		
70	220	-5.51		
60	220	-3		
50	220	-1.44		
40	220	-.567		
30	220	-.147		
20	220	+.002		
10	220	-		

Table 2 (Continued)

V	ω	10^{-19} x Determinant in 85 $r_s=1.2$	$r_s=1.7$	$r_s=3.2$
45	150	-.955	-1.94	-6.78
40	150	-.617	-1.26	-4.47
35	150	-.374	-.773	-2.79
30	150	-.21	-.439	-1.63
25	150	-.105	-.226	-.868
20	150	-.045	-.1	-.409
15	150	-.013	-.034	-.156
10	150	+.0006	-.003	-.029
30	75	-.059		
25	75	-.053		
20	75	-.031		
15	75	-.013		
10	75	-.0035		
5	75	-.0004		
12	37	$+7.5\times10^{-3}$		
11	37	$+3.8\times10^{-3}$		
10	37	$+1.6\times10^{-3}$		
9	37	$+.46\times10^{-3}$		
8	37	$-.08\times10^{-3}$		
7	37	$-.26\times10^{-3}$		
6	37	$-.26\times10^{-3}$		
5	37	$-.2\times10^{-3}$		
4	37	$-.12\times10^{-3}$		
3	37	$-.06\times10^{-3}$		

Table 3. $L_d = .08$, $L_q = .035$, $L_{SR} = .03$, $r_R = 0.6$, $J = 2.5x10^{-3}$, $B = .5x10^{-3}$

V	ω	10^{-19} x Determinant in 85 $r_s=1.2$	$r_s=1.7$	$r_s=3.2$
110	377	-16.3	-40.9	-182
100	377	-9.62	-26.3	-125
90	377	-4.79	-15.6	-82.6
80	377	-1.43	-7.91	-51.1
70	377	+.81	-	-28.6
60	377	+2.26	+.913	-12.8
50	377	+3.27	+3.36	-1.76
40	377	+4.29	+5.52	+6.92
30	377	+6.47	+9.2	+18.3
90	300	-9.45		
80	300	-5.59		
70	300	-2.95		
60	300	-1.24		
50	300	-.203		
40	300	+.397		
30	300	+.803		
20	300	+1.57		
70	220	-3.92		
60	220	-2.14		
50	220	-1.03		
40	220	-.397		
30	220	-.077		
20	220	+.073		
10	220	-		

Table 3 (Continued)

V	ω	10^{-19} x Determinant in 85 r_s=1.2	r_s=1.7	r_s=3.2
45	150	-.696	-1.43	-4.94
40	150	-.456	-.944	-3.38
35	150	-.281	-.592	-2.21
30	150	-.161	-.347	-1.36
25	150	-.082	-.185	-.778
20	150	-.035	-.085	-.396
15	150	-.009	-.028	-.159
10	150	-.006	-.005	-.007
30	75	-.032		
25	75	-.035		
20	75	-.023		
15	75	-.01		
10	75	-.003		
5	75	-.0003		
12	37	$+7.3\times10^{-3}$		
11	37	$+4\times10^{-3}$		
10	37	$+2\times10^{-3}$		
9	37	$+.8\times10^{-3}$		
8	37	$+.19\times10^{-3}$		
7	37	$-.08\times10^{-3}$		
6	37	$-.16\times10^{-3}$		
5	37	$-.15\times10^{-3}$		
4	37	$-.11\times10^{-3}$		
3	37	-		

Table 4. $L_d = .06$, $L_q = .035$, $L_{SR} = .03$, $r_R = 0.6$, $J = 2.5 \times 10^{-3}$, $B = 0.5 \times 10^{-3}$

V	ω	10^{-19} x Determinant in 85 $r_s=1.2$	$r_s=1.7$	$r_s=3.2$
110	377	-3.66	-18	-118
100	377	+.065	-9.45	-81.9
90	377	+2.98	-2.7	-52.5
80	377	+5.34	+2.73	-28.7
70	377	+7.48	+7.38	-8.85
60	377	+9.82	+12	+9.02
50	377	+13.1	+17.8	+28
40	377	+19.3	+27.6	+55.5
30	377	+39.3	+58.9	-
90	300	-4.45		
80	300	-2.29		
70	300	-.73		
60	300	+.39		
50	300	+1.27		
40	300	+2.17		
30	300	+3.67		
20	300	-		
70	220	-2.24		
60	220	-1.23		
50	220	-.56		
40	220	-.15		
30	220	+.12		
20	220	+.4		
10	220	-		

Table 4 (Continued)

V	ω	10^{-19} x Determinant in 85 $r_s=1.2$	$r_s=1.7$	$r_s=3.2$
45	150	-.43	-.92	-3.36
40	150	-.29	-.64	-2.52
35	150	-.19	-.42	-1.82
30	150	-.11	-.26	-1.25
25	150	-.056	-.15	- .78
20	150	-.019	-.064	- .41
15	150	+.008	-.003	- .11
10	150	+.04	+.063	-
30	75	-.012		
25	75	-.019		
20	75	-.014		
15	75	-.008		
10	75	-.003		
5	75	-.0001		
12	37	$+6.7\times10^{-3}$		
11	37	$+4\times10^{-3}$		
10	37	$+2.3\times10^{-3}$		
9	37	$+1.1\times10^{-3}$		
8	37	$+.46\times10^{-3}$		
7	37	$+.1\times10^{-3}$		
6	37	$-.07\times10^{-3}$		
5	37	$-.12\times10^{-3}$		
4	37	$-.1\times10^{-3}$		
3	37	-		

Table 5. $r_s = 1.2$, $L_d = .08$, $L_q = .035$, $L_{SR} = .03$

		10^{-19} x Determinant in 85			
V	ω	$r_R=0.6$ $J=2.5\times10^{-3}$ $B=0.5\times10^{-3}$	$r_R=1.2$ $J=2.5\times10^{-3}$ $B=0.5\times10^{-3}$	$r_R=0.6$ $J=5\times10^{-3}$ $B=0.5\times10^{-3}$	$r_R=0.6$ $J=2.5\times10^{-3}$ $B=10^{-3}$
110	377	-16.3	-23.4	-4.83	-3.07
100	377	-9.62	-14.5	-2.92	+1.98
90	377	-4.79	-7.68	-1.5	+5.39
80	377	-1.43	-2.48	-.46	+7.59
70	377	+.81	+1.46	+.29	+9
60	377	+2.26	+4.5	+.85	+10.1
50	377	+3.27	+7.1	+1.33	+11.6
40	377	+4.29	+9.96	+1.9	+15.5
30	377	+6.47	+15.2	+3.1	-
90	300	-9.45	-13.5	-2.78	-6.04
80	300	-5.59	-8.47	-1.7	-2.73
70	300	-2.95	-4.82	-.93	-.55
60	300	-1.24	-2.22	-.41	+.79
50	300	-.203	-.41	-.07	+1.57
40	300	+.397	+.87	+.16	+2.11
30	300	+.803	+1.94	+.35	+3
20	300	+1.57	+3.81	+.76	-
70	220	-3.92	-5.53	-1.16	-3.34
60	220	-2.14	-3.26	-.65	-1.7
50	220	-1.03	-1.74	-.33	-.68
40	220	-.397	-.76	-.14	-.13
30	220	-.077	-.18	-.03	+.15
20	220	+.073	+.19	+.03	+.33
10	220	-	-	-	-

Table 5 (Continued)

V	ω	10^{-19} x Determinant in 85 r_R=0.6 J=2.5x10^{-3} B=0.5x10^{-3}	r_R=1.2 J=2.5x10^{-3} B=0.5x10^{-3}	r_R=0.6 J=5x10^{-3} B=0.5x10^{-3}	r_R=0.6 J=2.5x10^{-3} B=10^{-3}
45	150	-.696	-1.06	-.21	-.65
40	150	-.456	-.74	-.14	-.41
35	150	-.281	-.49	-.09	-.25
30	150	-.161	-.31	-.05	-.13
25	150	-.082	-.18	-.03	-.058
20	150	-.035	-.085	-.01	-.015
15	150	-.009	-.024	-.004	+.01
10	150	-.006	-.017	+.003	+.034
30	75	-.032	-.055	-.026	-.031
25	75	-.035	-.064	-.017	-.034
20	75	-.023	-.046	-.009	-.022
15	75	-.01	-.024	-.004	-.01
10	75	-.003	-.009	-.001	-.027
5	75	-.0003	-.001	-.0001	-.001x10^{-3}
12	37	+7.3x10^{-3}	+16.1x10^{-3}	+.42x10^{-3}	+7.4x10^{-3}
11	37	+4x10^{-3}	+8.9x10^{-3}	+.1x10^{-3}	+4.1x10^{-3}
10	37	+2x10^{-3}	+4.3x10^{-3}	-.08x10^{-3}	+2.03x10^{-3}
9	37	+.8x10^{-3}	+1.59x10^{-3}	-.16x10^{-3}	+.8x10^{-3}
8	37	+.19x10^{-3}	+.09x10^{-3}	-.18x10^{-3}	+.2x10^{-3}
7	37	-.08x10^{-3}	-.59x10^{-3}	-.16x10^{-3}	-.06x10^{-3}
6	37	-.16x10^{-3}	-.79x10^{-3}	-.13x10^{-3}	-.14x10^{-3}
5	37	-.15x10^{-3}	-.09x10^{-3}	-.14x10^{-3}	-.13x10^{-3}
4	37	-.11x10^{-3}	-.06x10^{-3}	-	-.39x10^{-3}
3	37	-	-	-	-

Table 6. $r_S = 1.2$, $L_d = .08$, $L_q = .035$, $r_R = 0.6$, $J = 2.5x10^{-3}$, $B=0.5x10^{-3}$

V	ω	10^{-19} x Determinant in 85 $L_{SR}=.015$	$L_{SR}=.03$	$L_{SR}=.06$
110	377	-16.9	-16.3	-36.8
100	377	-10.1	-9.62	-23.2
90	377	-5.14	-4.79	-12.5
80	377	-1.59	-1.43	-3.98
70	377	+.86	+.81	+2.97
60	377	+2.49	+2.26	+9.29
50	377	+3.58	+3.27	+16.3
40	377	+4.47	+4.29	+26.9
30	377	+5.8	+6.47	+51.4
90	300	-9.8	-9.45	-20.4
80	300	-5.93	-5.59	-13
70	300	-3.22	-2.95	-7.52
60	300	-1.4	-1.24	-3.57
50	300	-.24	-.203	-.64
40	300	+.45	+.397	+1.8
30	300	+.89	+.803	+4.75
20	300	+1.42	+1.57	+12.9
70	220	-4.09	-3.92	-7.9
60	220	-2.3	-2.14	-4.71
50	220	-1.15	-1.03	-2.55
40	220	-.47	-.397	-1.17
30	220	-.097	-.077	-.29
20	220	+.087	+.073	+.41
10	220	-	-	-

Table 6 (Continued)

V	ω	10^{-19} x Determinant in 85 L_{SR}=.015	L_{SR}=.03	L_{SR}=.06
45	150	-.76	-.696	-1.35
40	150	-.51	-.456	-.96
35	150	-.32	-.281	-.65
30	150	-.19	-.161	-.42
25	150	-.1	-.082	-.25
20	150	-.046	-.035	-.13
15	150	-.012	-.009	-.039
10	150	+.008	-.006	-.038
30	75	-.052	-.032	+.16
25	75	-.048	-.035	+.017
20	75	-.03	-.023	-.022
15	75	-.014	-.01	-.021
10	75	-.005	-.003	-.011
5	75	-.0006	-.0003	-.002
12	37	$+7.6\times10^{-3}$	$+7.3\times10^{-3}$	+.053
11	37	$+4\times10^{-3}$	$+4\times10^{-3}$	+.034
10	37	$+1.7\times10^{-3}$	$+2\times10^{-3}$	+.02
9	37	$+.48\times10^{-3}$	$+.8\times10^{-3}$	+.012
8	37	$-.14\times10^{-3}$	$+.19\times10^{-3}$	+.006
7	37	$-.38\times10^{-3}$	$-.08\times10^{-3}$	+.003
6	37	$-.4\times10^{-3}$	$-.16\times10^{-3}$	$+.7\times10^{-3}$
5	37	$-.33\times10^{-3}$	$-.15\times10^{-3}$	$-.14\times10^{-3}$
4	37	-	$-.11\times10^{-3}$	$-.22\times10^{-3}$
3	37	-	-	-

The amount of information in Tables 1 through 6 is extensive, and considerable study of them is required to draw proper conclusions. Because of the non-linear nature of the motor, it is necessary to examine the effects of changing parameters at each particular operating point to predict with certainty the effect on stability when certain parameters are varied.

The following observations are made as a result of the computer calculations:

1. In the range of 40% to 100% speed
   a. reducing the speed decreases the possibility for asymptotic stability,
   b. reducing voltage increases the prospects for stability,
   c. adding resistance in series with the stator lines to the motor makes asymptotic stability less likely,
   d. reducing L_d alone improves the prospects for stability. (This is believed to represent the effect of saturation. Since the equivalent air gap for L_d is small, this parameter is reduced more by saturation than the other machine inductances.),
   e. two-to-one changes in either of the constants r_R or L_{SR} do not effect materially the range of voltage for stability at a given speed,

f. increasing either J or B two-to-one tends to improve stability with the increase in B having the greater effect.

2. Below 40% speed the range of stability is a more complex function of the system parameters, and it is necessary to examine parameter variations about each operating point to ascertain the direction to go with any given parameter to improve stability.

IV. EXPERIMENTAL RESULTS

A. Determination of Constants

1. B

The viscous friction coefficient B was determined by measuring the mechanical time constant of the motor. A photograph of the motor used throughout the experimental work is shown in Fig. 3. This is a reluctance synchronous motor rated $\frac{1}{4}$ horsepower, 110 volts, 3 phase, 1800 RPM which was provided without charge by the General Electric Company, Fort Wayne, Indiana. It contains a squirrel cage induction motor type rotor which has flats machined on it to provide reluctance motor action. The rotor of the machine is shown in Fig. 4.

The mechanical time constant was determined by measuring the slow down time of the motor. With the motor running at a steady-state speed, the power applied to the motor was switched off and the time for the motor to slow down to 37% of its initial speed was measured with a stop watch. The initial speed was measured with a Hasler Tachometer serial #81072. A general Radio Strobotac Type No. 631-B set for 37% of the initial speed and directed toward the shaft keyway was used to sense the final speed. Table 7 shows the results of several tests.

Fig. 3. General Electric model no. 5SK43 MG reluctance synchronous motor $\frac{1}{4}$ horsepower, 110 volt, 3 phase, 1800 RPM

Fig. 4. Rotor

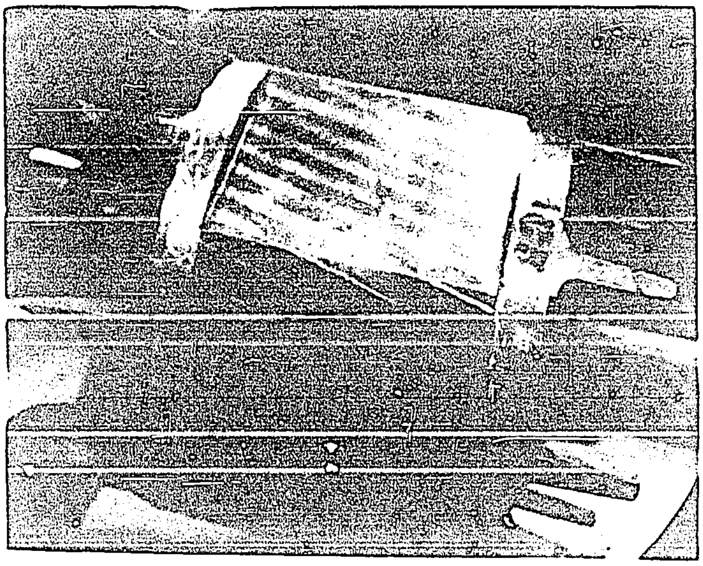


Table 7. Mechanical time constant measurements

Initial RPM	Final RPM	Time in seconds to slow down from initial to final speed
1800	660	7.3
1800	660	6.9
1800	660	7.3
1800	660	7.1
900	330	5.1
900	330	5.2
900	330	5.2
900	330	4.8

As shown in Table 7, the tests gave reasonably consistent results. The power to the motor was interrupted either by switching off the 400 cps supply to the SCR circuit which was used as the variable frequency power supply for the motor, or by opening a switch directly in series with the motor leads. Several trials indicated that either way of interrupting the power gave essentially the same results.

After the power to the motor is interrupted, the differential equation for the motor is

$$J \frac{d\omega}{dt} + B = 0 \qquad (102)$$

It is assumed that the motor has only inertia and viscous friction loads. The solution to 102 is

$$\omega = \omega_o \, e^{-B/J \; t} \tag{103}$$

Thus, the mechanical time constant of the motor is J/B. The rotor inertia given by the motor manufacturer is

$$J = 8.6 \# \text{ in}^2 \quad \text{or} \quad 2.5 \times 10^{-3} \text{ kg-meter}^2 \tag{104}$$

If Equation 103 exactly represented the motor slow down operation, the time to slow down would have been the same for the two different initial speeds in Table 7. An approximate average mechanical time constant from Table 7 is used to calculate B.

$$\frac{J}{B} = 6.25 \text{ seconds} \tag{105}$$

$$B = \frac{J}{6.25} = \frac{2.5\text{x}10^{-3}}{6.25} = 0.4\text{x}10^{-3} \text{ kg-m}^2/\text{sec} \tag{106}$$

The time constant used in the calculation of B is within 20% of the actual measured values over the speed range concerned. The fact that there is apparently more damping at lower speeds could be the result of an increased bearing friction, or an increased damping action of the squirrel cage rotor winding at lower speeds. In any event, the accuracy of the assumed representation is believed to be sufficient to get useful practical results from the stability analysis.

2. L_d and L_q

Although several means for determining the direct-axis and quadrature-axis inductances were explored, the most accurate approach for the motor used in this work is believed to be the slip test (54, pp. 51-52). The motor of Fig. 3 was mechanically coupled to a d-c drive motor

Kimble Electric Company
Compound DC Motor Ser. No. 140081
230 volt, 5 HP, 1150 RPM

The following M-G set was used to deliver electrical power to the stator windings of the test motor:

General Electric Company
DC Shunt Motor Model No. 33A602
110 volt, 15 HP, 750/2250 RPM

General Electric Company
AC Generator No. 5237269
220 volt, 3 phase, 60 cps, 10 KW

During the test a 0.1 ohm load rack was connected in series with each line to the test motor. A Tektronix Type 551 dual beam oscilloscope was used to record simultaneously the line-to-neutral voltage and the voltage drop across the 0.1 ohm resistor in series with one stator lead to the reluctance motor. The test procedure was to set the drive motor at a given speed, adjust the speed of the M-G set electrical supply approximately to the same value, and then record the peak-to-peak maximum and minimum amplitudes of both scope traces. The direct-axis reactance x_d is determined from the maximum voltage divided by the minimum current. x_q is calculated by dividing the minimum voltage by the maximum current. Table 8 gives the test results.

Table 8. Slip test data

M-G set RPM	Reluctance motor line	Peak-to-peak line to neutral volts Maximum	Minimum	Peak-to-peak line amperes Maximum	Minimum	Calculated x_d/x_q
1420	Blue	25	20	3.6	1.6	2.81
1420	Blue	70	62	9.4	4	2.66
1420	Orange	74	62	0.2	3.8	2.89
1210	Orange	66	56	9.4	3.8	2.92
1210	Orange	33	28	5	2	2.95
1210	Blue	32	28	5	2	2.86
1210	Blue	60	52	9.2	3.6	2.95

As shown, the voltage applied to the test motor was adjusted to several different values and data was taken at two different speeds. The speed range was limited to that shown by the drive motor.

The major sources of error in the calculated results arise from the error in accurately observing the deflection of the oscilloscope traces, neglecting the core loss, and neglecting the resistance of the reluctance motor windings. The oscilloscope was calibrated carefully before the tests and the resistances of the 0.1 ohm load racks were checked with a resistance bridge. Although the calculated results are quite consistent, since the above sources of error are present, it is recommended generally (54) that the slip test be used to establish only the x_d/x_q ratio. Another means should be used to determine the actual direct-axis reactance. In this case, a reasonably accurate value of x_d is obtained from no load test data. With no load and a small winding resistance, the torque angle is approximately zero, i_{qo} is nearly zero and i_{do} is close to the crest value of the a-c line current. Equation 88 then reduces to

$$L_d \approx \frac{V}{\omega_o I} \tag{107}$$

Table 9 gives the results of several no load tests on the motor of Fig. 3 to determine L_d. Some of this data was taken by the motor manufacturer and some by the author with the motor supplied from a variable speed M-G set or a 60 cps 3 phase laboratory supply.

Table 9. No load tests

Line to neutral volts crest	Line amperes crest	RPM	Calculated L_d in millihenries using 107
90	2.26	1800	105.7
100	2.7	1800	98.7
20.7	1.2	725	113.5
37.8	2.4	725	103.5
45.7	1.13	1820	106
83.7	2.04	1820	107.3

From Table 8 and Table 9 and rounding off to reasonably even numbers the following values are selected for the test motor

L_d = 105 millihenries (108)

L_q = 35 millihenries (109)

3. r_R

The equivalent rotor resistance is one of the more difficult constants to determine as it cannot be measured directly. The most reasonable approximation is considered to be the value calculated from locked rotor watts data. Table 10 shows the results of locked rotor tests by the manufacturer and by the author.

Table 10. Locked rotor tests

RMS line amperes	Total watts	Calculated watts loss in stator windings & measuring circuits	Calculated ohms rotor resistance per phase
13.5	1350	657	1.27
14.9	1470	798	1.31

The fact that the core loss is neglected and ignoring the shunting effect of the stator to rotor mutual reactance, are the two principal sources of error in this approach to finding the equivalent rotor resistance. However, these two factors tend to produce errors in the opposite direction.

4. Summary

To this point, all of the constants for the motor are known except for the rotor self inductance L_{RR} and the stator to rotor mutual inductance L_{SR}. These parameters are also extremely difficult to determine as they cannot be directly measured. Therefore, values are assumed for these constants, which are believed to provide reasonably accurate results for the stability analysis. The value of L_{RR} assumed is

$$L_{RR} = \frac{3}{2} L_{SR} \qquad (110)$$

Referring to the direct-axis equivalent circuit in Fig. 1, the value of L_{RR} in 110 is equivalent to neglecting the leakage reactance in the rotor circuit. This is believed to be quite a good assumption as the rotor squirrel cage is actually many single turn windings in parallel which should have quite low reactance.

The precise value of L_{SR} to be used is not known. However, several different values were used in the computer calculations leading up to the information in Tables 1-6. As a result, it was found that this parameter could be varied over quite a wide range without materially effecting the stability results. Thus, a value of in the order of

$$L_{SR} = 0.5\ L_d \qquad (111)$$

is considered a reasonable approximation. Again referring to Fig. 1, this is equivalent to assuming 50% leakage reactance in the direct-axis circuit.

The motor constants are summarized below:

$$B = 0.4\text{x}10^{-3}\ \text{kg-m}^2/\text{sec (calculated from tests)} \qquad (112)$$

$J = 2.5\text{x}10^{-3}$ kg-meter^2 (from manufacturer) (113)

L_d= 105 millihenries (calculated from tests) (114)

L_q= 35 millihenries (calculated from tests) (115)

L_{SR}= 50 millihenries (assumed) (116)

$L_{RR}= \frac{3}{2} L_{SR}$ = 75 millihenries (assumed) (117)

r_S= 1.2 ohms (from manufacturer) (118)

r_R= 1.2 ohms (calculated from tests) (119)

The computer calculations tabulated in Tables 1-6 show the effect on stability of changing the above parameters.

B. M-G Set Results

In the analysis of Section III, one of the assumptions is that the applied voltage to the motor is a balanced sinusoidal 3-phase source. A real source which is a practical realization of this assumption is a 3-phase alternator. Thus, in these tests a relatively large d-c motor driven alternator was used to supply the test motor. The name-plate information for the M-G set is listed below:

General Electric Company
DC Shunt Motor Model No. 33A602
110 volt, 15 HP, 750/2250 RPM

General Electric Company
AC Generator No. 5237269
220 volt, 3 phase, 60 cps, 10KW

Since the KVA rating of the alternator is large relative to the rating of the test machine, regulation effects are minimized. The test procedure was to operate the M-G set at different speeds with different 3-phase output voltages and with different values of resistance in series with the alternator output lines to the test motor. The operating conditions where hunting occurred were observed and the values of speed, voltage, and current were recorded for several different stable operating situations. The calculated voltage V is the crest value of the average of the three line to neutral voltages. The line to neutral circuit resistance was measured with a resistance bridge and the average of the three bridged values was 1.7 ohms with no load racks in series with the test motor, 3.2 ohms with the 1.5 ohm load racks, and 7.5 ohms with the 6 ohm load racks.

The trends in stability performance as the system parameters are varied correlate quite well when Table 11 is compared with the computer calculations. The best agreement is in comparing Table 11 and Table 4. This implies that L_d is lower than 105 millihenries which is believed to be reasonable because of the effect of saturation of the motor. In general the calculations predict asymptotic stability less often than it occurs with the actual system. This is as it should be since the stability criteria being used in the analysis is a sufficient condition for asymptotic stability. At the lower frequency with resistors added in series with the motor, there is the least precise correlation between the observed and predicted performance. The major factors which cause the difference between the performance of the

Table 11. M-G set test results

RPM	Load rack ohms in series with each line	Test motor line-to-neutral RMS volts A	B	C	Test motor RMS line amperes A	B	C	Calculated V
725	0	14.7	14.3	15	0.82	0.85	0.9	20.8
725	0	26.8	26.0	27.4	1.6	1.73	1.77	37.8
	Note: There is no hunting at either of the above points and for any voltage in between those shown.							
1820	0	32.2	32.2	32.5	0.75	0.8	0.83	45.7
1820	0	58.9	58.9	59.7	1.36	1.53	1.53	83.7
	Note: Again there is no hunting at either of the above points and for any voltage in between those shown.							
735	1.5	17.8	17.8	18.2	1.0	1.03	1.06	25.5
735	1.5	30.4	30.4	30.9	1.95	2.03	2.07	44.3
	Note: There is no hunting at either of the above points, but there is hunting for all voltages in between those shown.							
1800	1.5	33	33	33.5	0.7	0.77	0.8	46.9
1800	1.5	57	57	58	1.33	1.47	1.5	81.1
	Note: There is no hunting at either of the above points and for any voltage in between.							
740	6	29.6	29.8	30.3	1.6	1.64	1.72	42.3
	Note: There is no hunting at this point, but hunting does occur for voltages slightly below that shown.							
1800	6	30.2	30.3	30.8	0.6	0.65	0.7	43.1
1800	6	56.4	56.7	57.8	1.3	1.4	1.45	83.3
	Note: There is no hunting at these two points nor at any voltage in between.							

motor supplied from the M-G set and that predicted by the computer data in Tables 1-6 are believed to be the following:

1. The precise values of certain of the system constants are not known. However, the values used have been at least partially verified by test with the exception of L_{SR}, which can be changed over quite a wide range without materially effecting stability.
2. Saturation of the machine will tend to reduce the motor inductances at higher voltages for a given frequency. This means that the machine inductances vary with the applied voltage, with the greatest variation occurring in L_d.
3. The computer data only indicates where asymptotic stability will occur. It does not necessarily imply hunting when the determinant in 85 is not positive.

C. SCR Power Supply

1. Power circuit

The power circuit for the SCR supply is shown in Fig. 5. The components shown are described below:

F1-F12 Bussman KAA20 fuse

L1-L3 Air core inductor 8 millihenries/2 millihenries (4 100 turn single layer windings #14 wire, two windings in series each side of center-tap, 4" I.D., 4 3/4" O.D., 8" long)

M General Electric 4SK43MG reluctance synchronous motor

R1-R3 1000 ohm 10 watt damping resistor

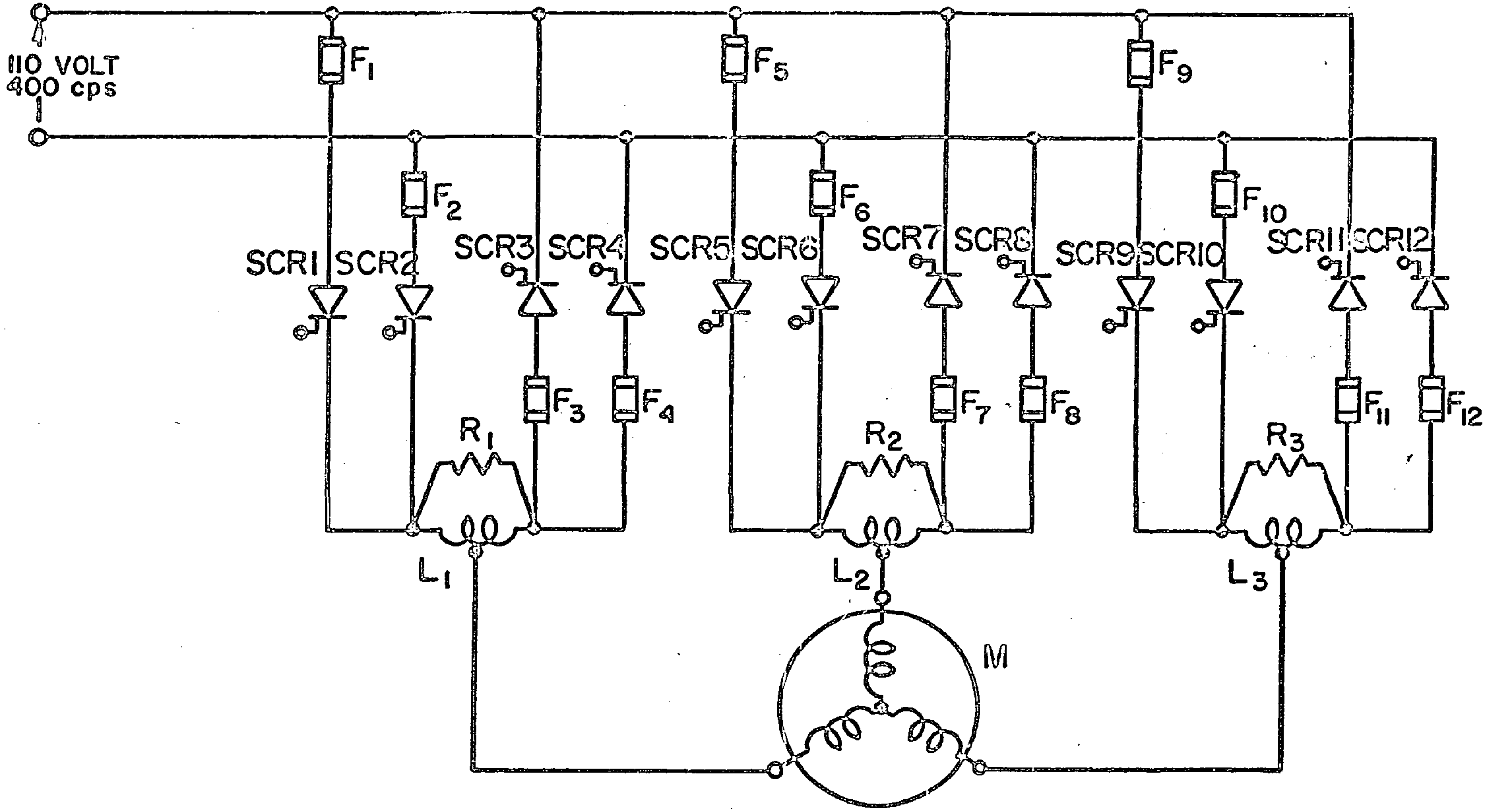


FIG. 5 — SCR MODEL POWER CIRCUIT

SCR1-SCR12 General Electric silicon controlled rectifier 2N689, 500 volt, 35 amperes RMS (mounted on 6x6x1/16" copper heat sink)

Fig. 5 is a single phase to three phase cycloconverter that delivers an adjustable voltage, adjustable frequency output from a 400 cps source. It was selected for this research because it is used in several important practical applications (37), (50, Chapters 3 and 11), (55, 56, 57, and 58). As the cycloconverter is a line commutated inverter, no additional circuit components are required to provide for SCR commutation. Thus, it is one of the simplest and most reliable inverter arrangements.

A brief description of the operation of the circuit in Fig. 5 is as follows. Considering SCR1 through SCR4, on a positive half cycle of the motor frequency when the motor line connected to L1 is to be positive, SCR1 and SCR2 alternately rectify the 400 cps source to deliver a unidirectional positive voltage to the motor line. On the negative half cycle of the motor frequency, SCR3 and SCR4 are alternately gated on to produce a negative average voltage on the motor line during this interval. Inductor L1 does not play a fundamental role in this basic operation. In fact with a purely resistive load, inductors L1 - L2 would not be required. However, for any inductive load, which is the nature of the load represented by the motor, these inductors are used to assure reliable commutation of the SCR's. Their main function is performed during the transient period when the motor line voltage is being reversed. If SCR3 and SCR4 are gated on after SCR1 and SCR2 have been alternately rectifying power to the motor, SCR1 and SCR2 will continue to carry the motor current for a time since the motor inductance will maintain current flow into the motor line. Thus, there is an interval during which SCR3 and SCR4 are alternately gated on while SCR1 and SCR2 must also still be ready to conduct motor current as required. Inductor L1 limits the circulating current that can flow through SCR1 and SCR4 or through SCR2 and SCR3 when these pairs of SCR's are on simultaneously. Thus, when the polarity of the motor voltage is to be reversed, SCR3 and SCR4 are alternately gated on to operate as rectifiers and at the same time the firing angle of SCR1 and SCR2 is delayed so these devices can operate in an inverting mode to continue to conduct current into the motor line for a time when the motor line voltage is negative. It is extremely important not to delay the firing of SCR1 and SCR2 for a full 180^{o} or commutation failure may result during the period in which the motor line voltage is reversed. The SCR's connected to L2 and L3 operate in a similar manner with 120^{o} phase displacement so as to provide 3 phase voltage to the motor.

The detailed waveforms of the circuit of Fig. 5 are relatively complicated and will not be discussed here. In a study of these waveforms, it is generally most clear to use a center-tap of the single phase source as the voltage reference. Waveforms for similar line commutated inverters are discussed at some length in reference (50, Chapter 3).

2. Control

A detailed block diagram of the SCR control is given in Fig. 6. Circuit diagrams of each of the blocks are shown in Fig. 7 through Fig. 13, except the diode logic circuits are included in the select amplifier circuit diagram. On all of the circuit diagrams the watt rating of resistors is understood to be $\frac{1}{2}$ watt, resistance values are in ohms, and capacitance values are in microfarads unless otherwise noted.

There are two independent control inputs. The magnitude of the 30 volt adjustable d-c supply in Fig. 7 determines the voltage that will be delivered from the SCR circuit to the motor, and the potentiometer in Fig. 8 establishes the motor frequency. The differential input amplifier does not play an essential role in the overall operation. It is a conventional differential amplifier (59) to permit establishing the signal input voltage level at a more convenient value than that required to drive the select amplifiers.

The master oscillator of Fig. 8 is a unijunction transistor oscillator and pulse amplifier (60). Its output is a 10 volt supply which drops to practically zero for about 50 microseconds every 1/6 of a cycle of the motor frequency. This

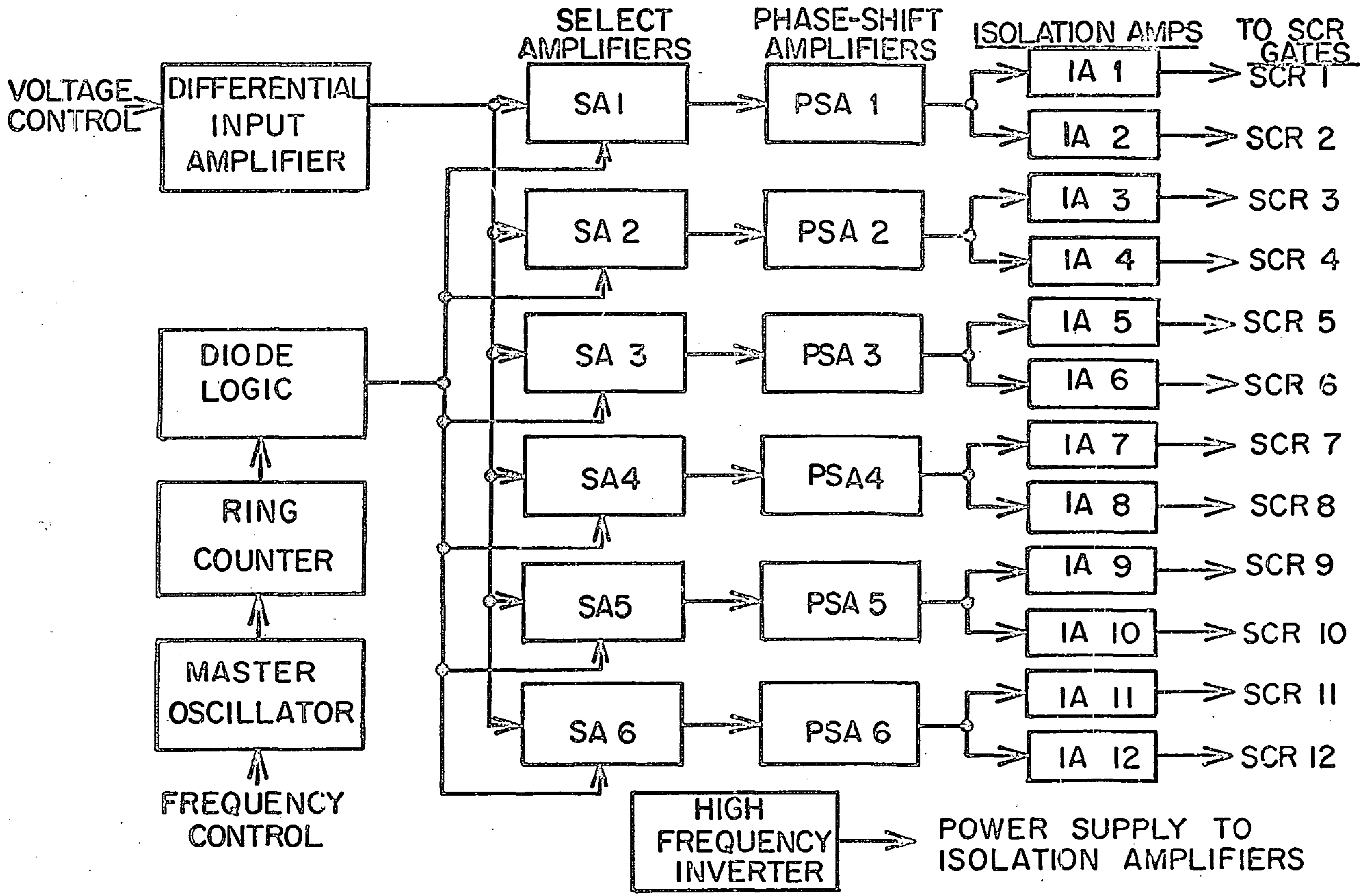


FIG. 6 — DETAILED BLOCK DIAGRAM OF SCR CONTROL

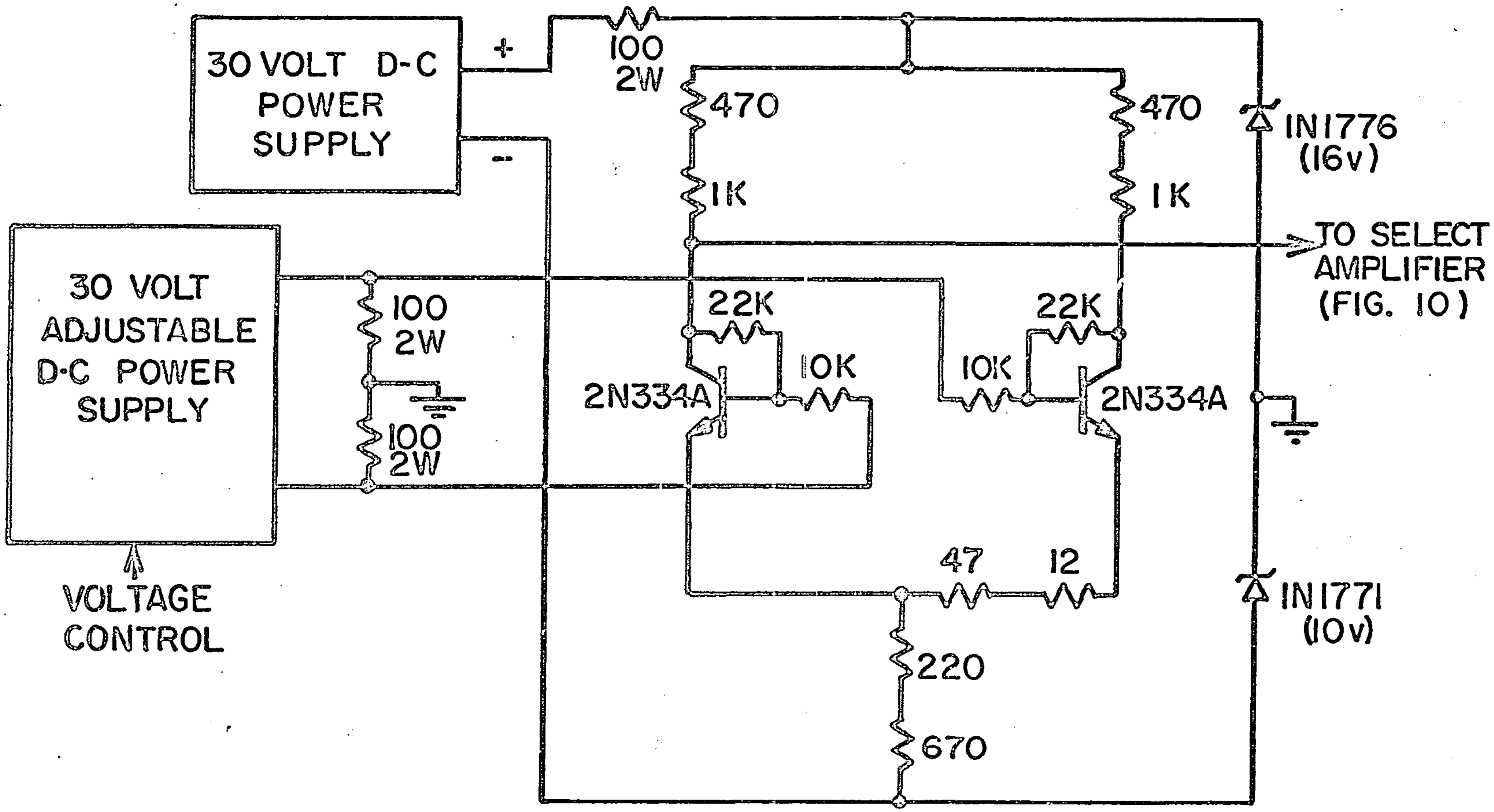


FIG. 7— DIFFERENTIAL INPUT AMPLIFIER

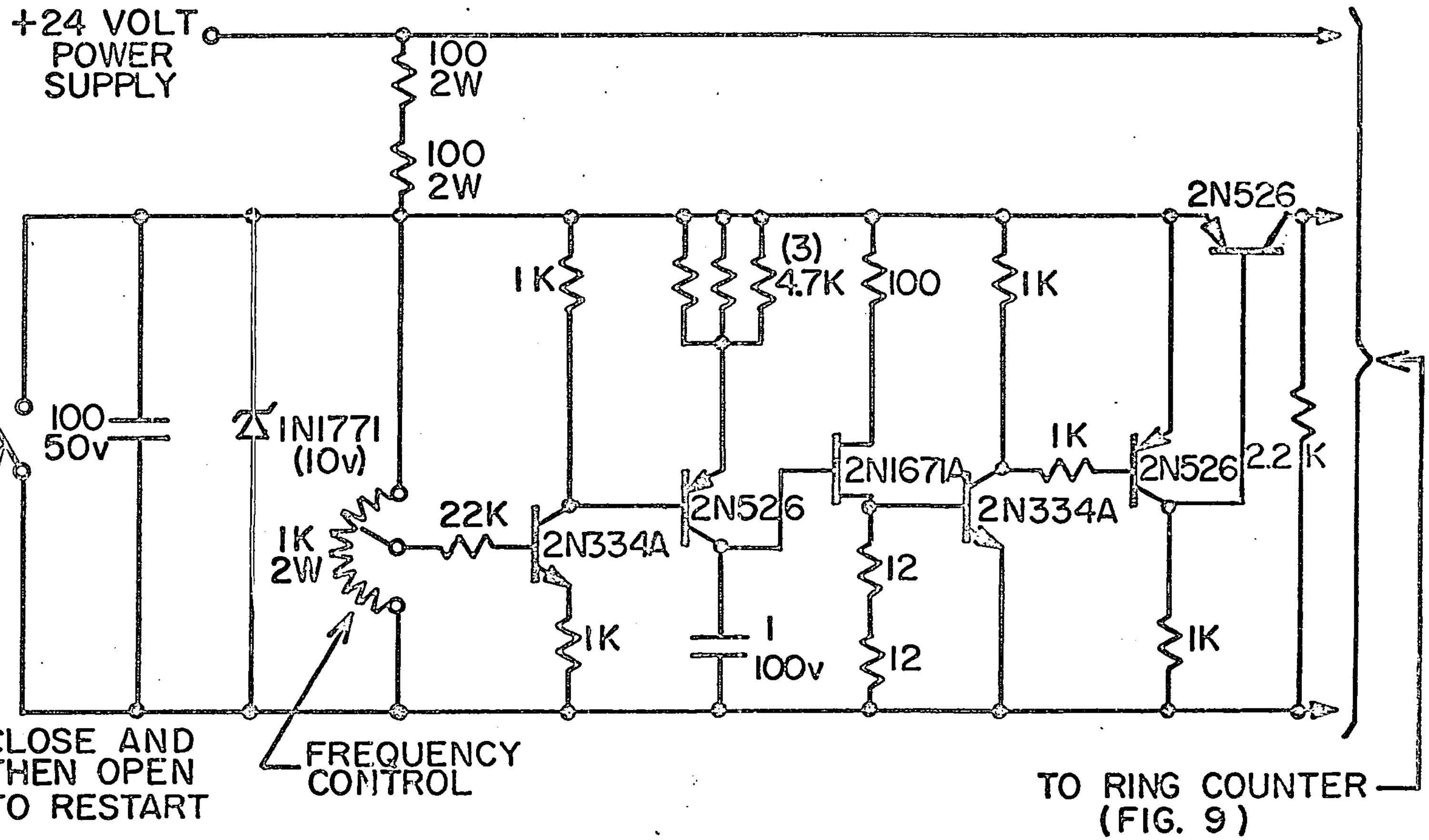


FIG. 8 — MASTER OSCILLATOR

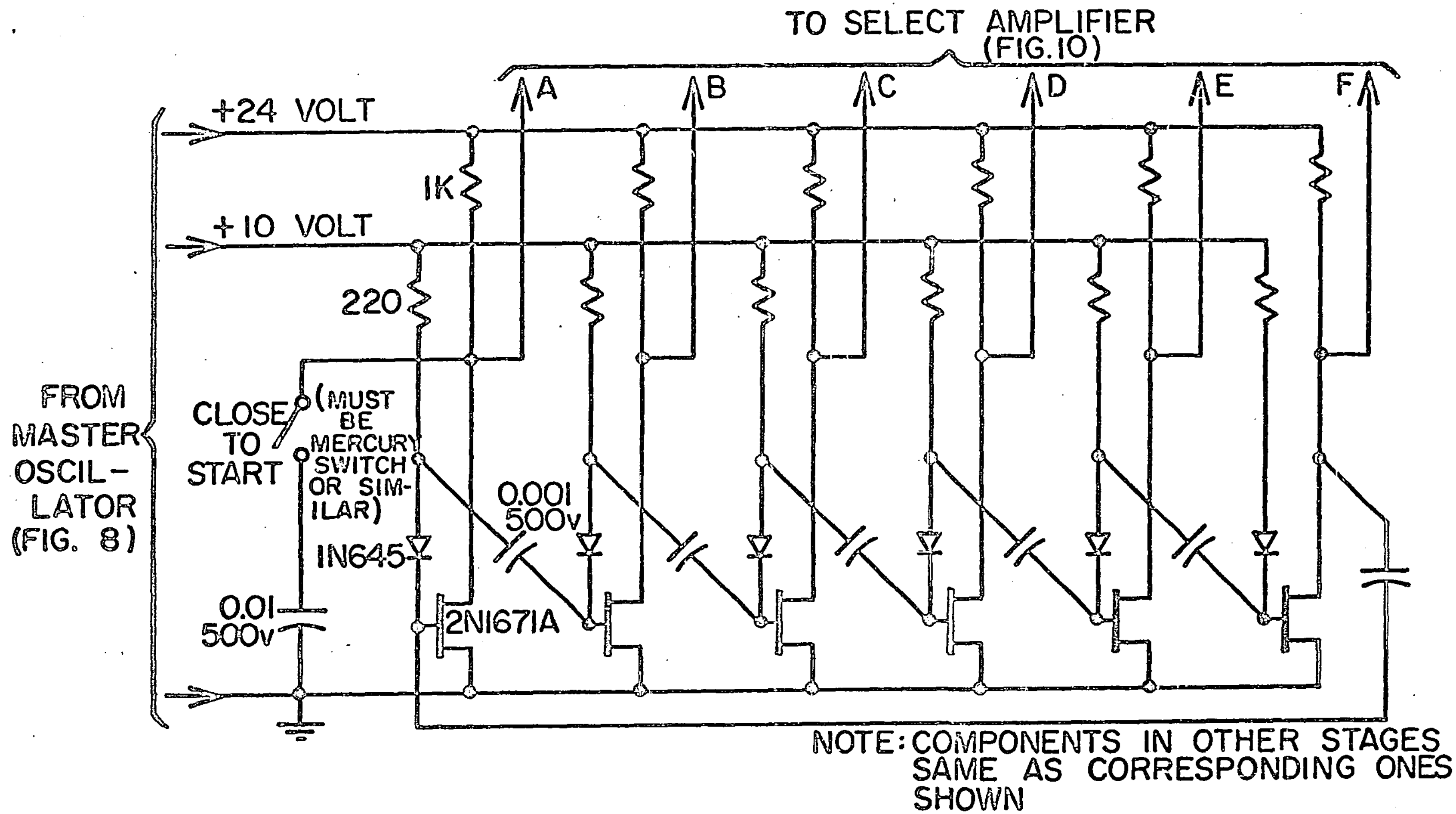


FIG. 9— RING COUNTER

NOTE: ALL DASHED LINES ARE NOT REPEATED FOR OTHER SELECT AMPLIFIERS AND AND ARE CONNECTED AT POINTS MARKED ✳ TO CORRESPONDING POINTS IN OTHER AMPLIFIERS

SELECT AMPLIFIER	CONNECTED TO RING COUNTER
SA 1	ABC
SA 2	DEF
SA 3	CDE
SA 4	FAB
SA 5	EFP
SA 6	BCD

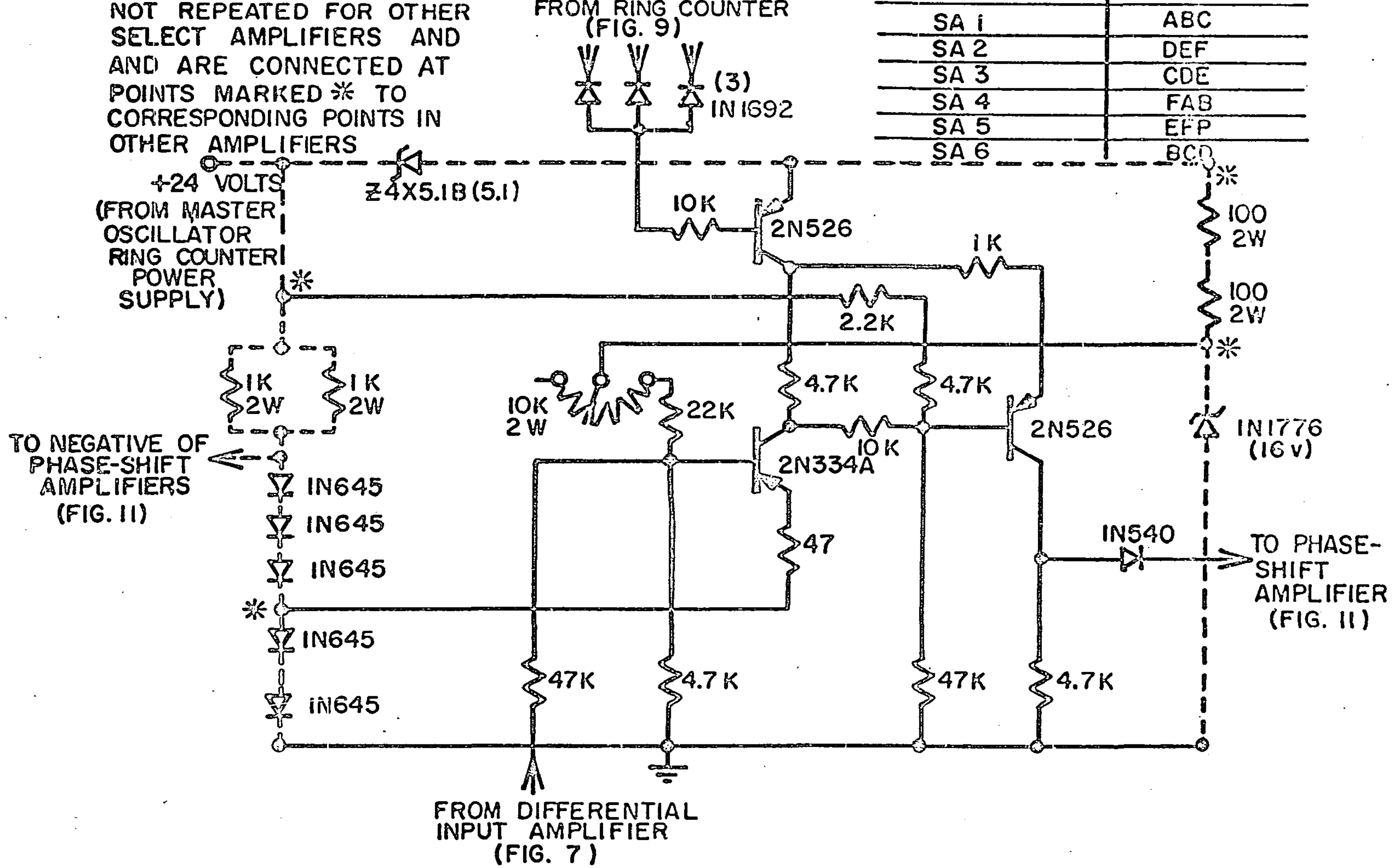
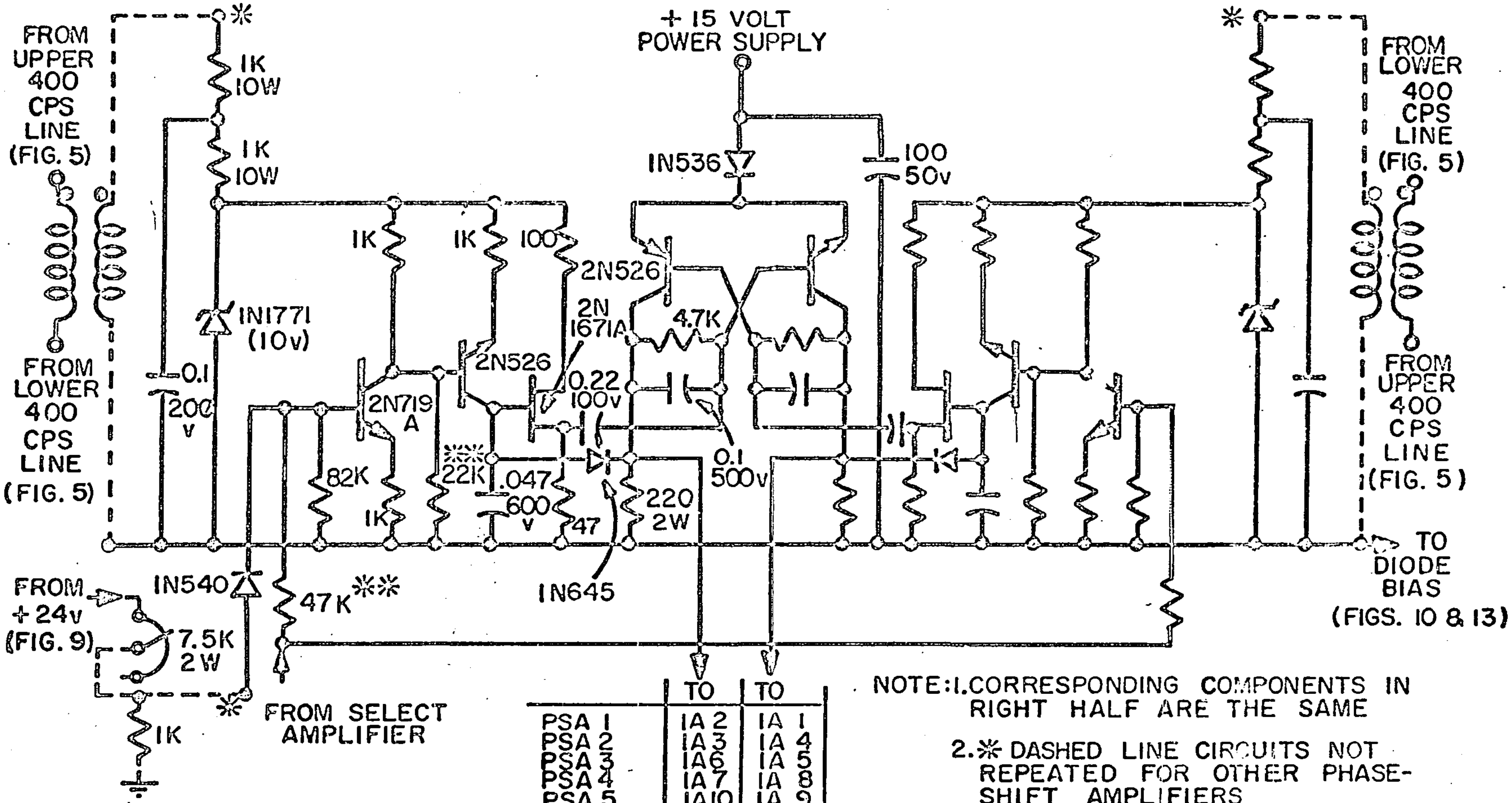


FIG. 10 — SELECT AMPLIFIER



	TO	TO
PSA 1	IA 2	IA 1
PSA 2	IA 3	IA 4
PSA 3	IA 6	IA 5
PSA 4	IA 7	IA 8
PSA 5	IA 10	IA 9
PSA 6	IA 11	IA 12

NOTE: 1. CORRESPONDING COMPONENTS IN RIGHT HALF ARE THE SAME

2. ※ DASHED LINE CIRCUITS NOT REPEATED FOR OTHER PHASE-SHIFT AMPLIFIERS

3. ※※ VALUES VARY AS SELECTED TO MATCH CIRCUITS

FIG. 11 — PHASE-SHIFT AMPLIFIER

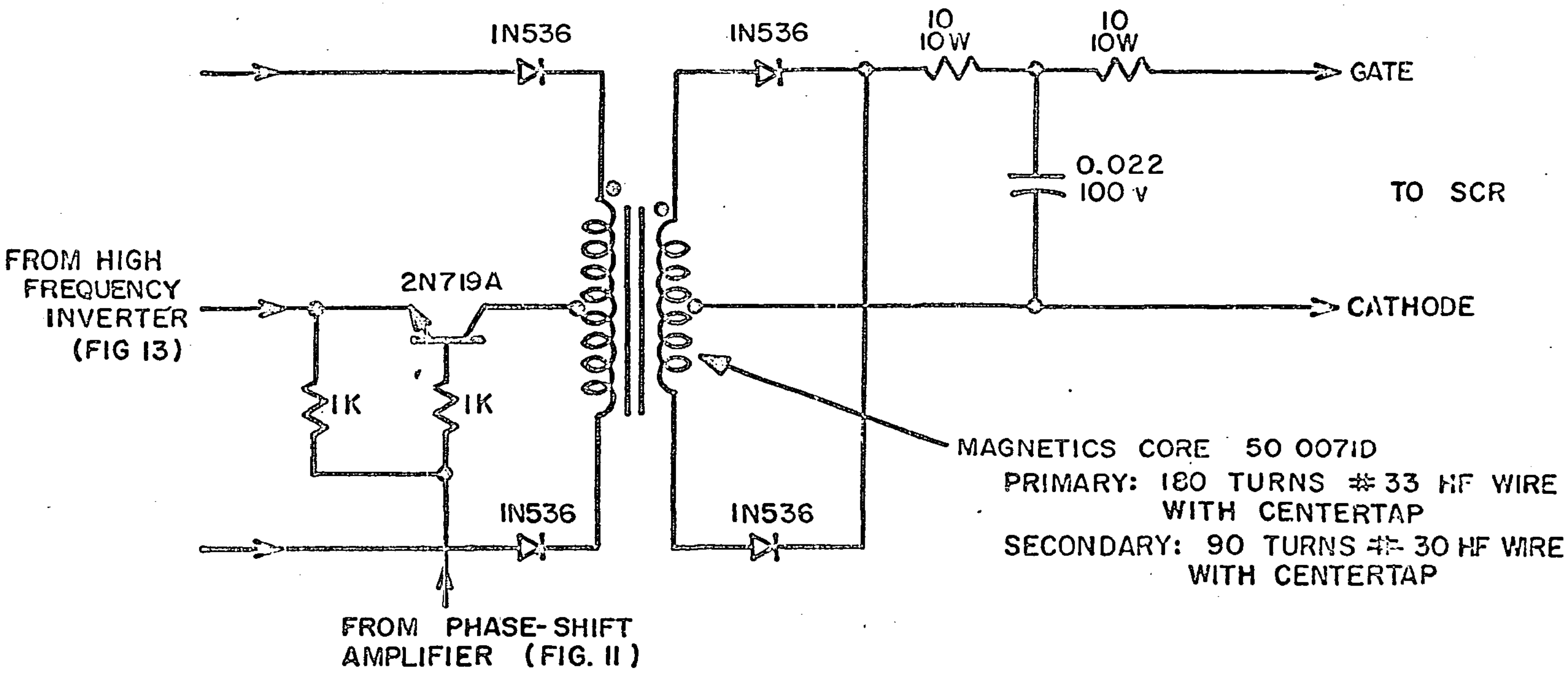


FIG. 12 — ISOLATION AMPLIFIER

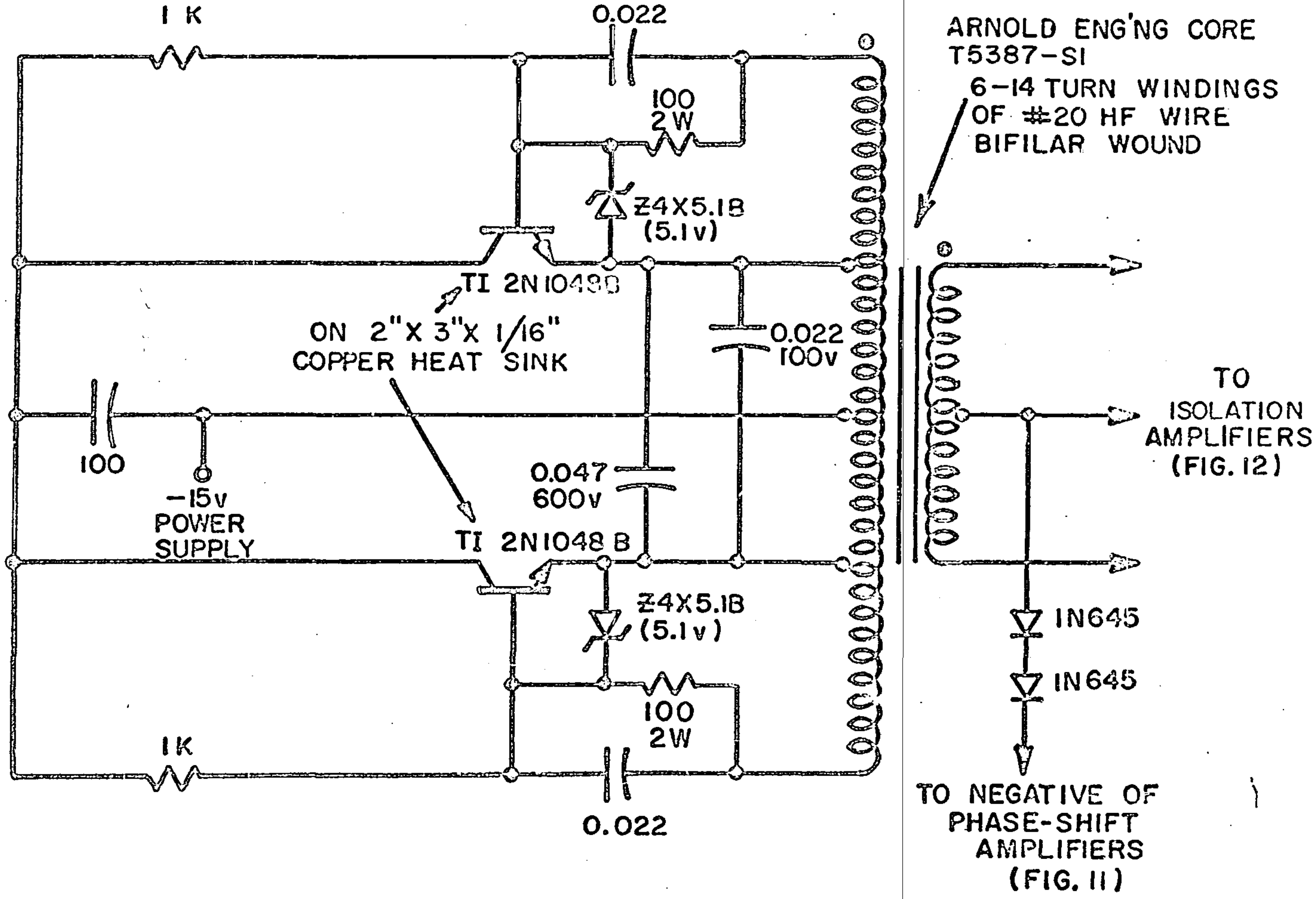


FIG. 13— HIGH FREQUENCY INVERTER

10 volt supply triggers the ring counter of Fig. 9 to shift it from one stage to the next. The ring counter is the type described in Schwartz (61, p. 7-35). When a given stage of the ring counter is on, the particular unijunction has a relatively low voltage across it so the point A, B, C, D, E, or F is pulled down toward ground. This in turn biases on the upper 2N526 of Fig. 10. Therefore, the select amplifier is operating only when a ring counter stage which is on, is connected to the base of the switching transistor at the top of Fig. 10. When a select amplifier is operating, it amplifies the voltage from the differential input amplifier and supplies this signal to the phase-shift amplifier Fig. 11. Fig. 11 is a conventional unijunction delay circuit to trigger a flip-flop a certain time after the zero crossing of the 400 cps wave (60, 62). The output of the flip-flop of Fig. 11 controls the transistor of Fig. 12 to energize the gate-cathode of a particular SCR. The isolation amplifier supplied from the transistor inverter of Fig. 13 provides a means to isolate electrically the SCR gate-cathode from the control circuits.[1] In summary, the voltage control acts through the select amplifier, phase shift amplifier, and isolation amplifier to determine the firing angle of the SCR's.

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[1]This means of providing isolation was conceived and reduced to practice by B. D. Bedford and F. G. Turnbull of General Electric in 1963.

The master oscillator and ring counter act through the diode logic to determine which select amplifiers are in operation and thus to determine which SCR's receive the voltage control signal. When a select amplifier is not in operation, its particular phase shift amplifier does not trigger the flip-flop of Fig. 11 until late in the 400 cps half cycle as determined by the network in the lower left corner of Fig. 11. This network sets the inverter firing angle of the SCR's.

Photographs of the more important waveforms for the SCR supply are shown in Fig. 14 through Fig. 27. All of these photographs were taken with the complete SCR power supply in operation and connected to the motor of Fig. 3.

3. Stability test results

For these tests, the reluctance motor was operated from the SCR power supply. The wattmeters and ammeters used for the M-G set tests Table 11 were included in the motor circuit to keep the circuit resistances the same. However, these instruments were not used for measurements as they are not accurate because of the harmonics which are present. A Hewlett Packard Model 302A Wave Analyzer was used to measure the line to neutral voltage on the motor. The test procedure was to operate the motor at different speeds by adjusting the frequency control, different voltages as established by the voltage control, and with different added resistances in series with the motor lines. The conditions where hunting

Fig. 14. 2 ms/cm, 10v/cm both traces

Upper - 10 volt supply to ring counter Fig. 9

Lower - voltage across 1μf capacitor in master oscillator Fig. 8

Fig. 15. 2 ms/cm, 10 v/cm both traces

Upper - voltage across sixth unijunction transistor in ring counter Fig. 9

Lower - voltage across first unijunction transistor in ring counter Fig. 9

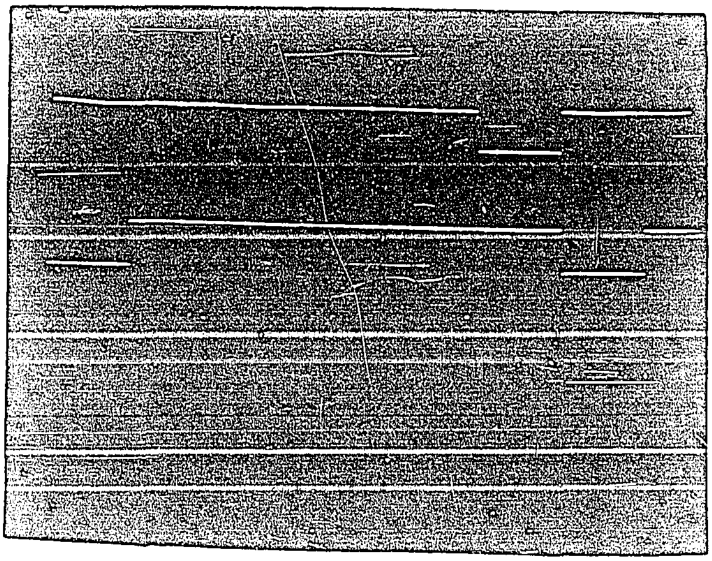


Fig. 16. 2 ms/cm, 10 v/cm both traces

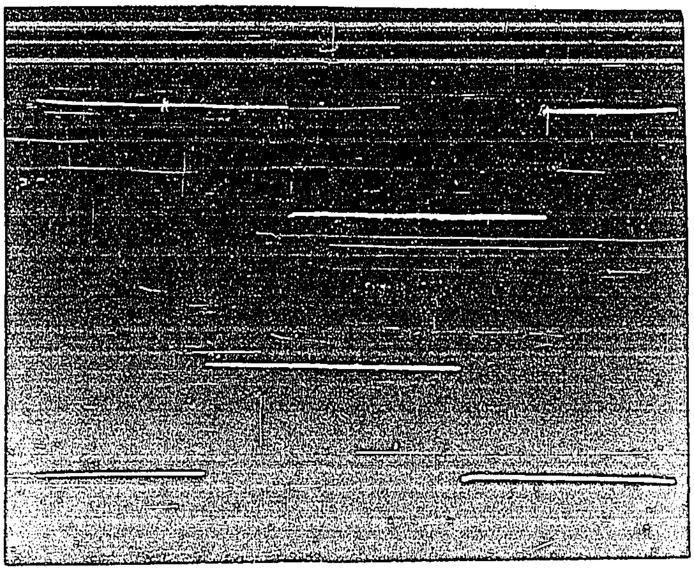
Upper - collector to ground voltage of 2N526 transistor Fig. 10 driven from ring counter (select amplifier #1)

Lower - same (select amplifier #3)

Fig. 17. 2 ms/cm, 10 v/cm both traces

Upper - collector to ground voltage of output transistor in select amplifier #1

Lower - same for select amplifier #3



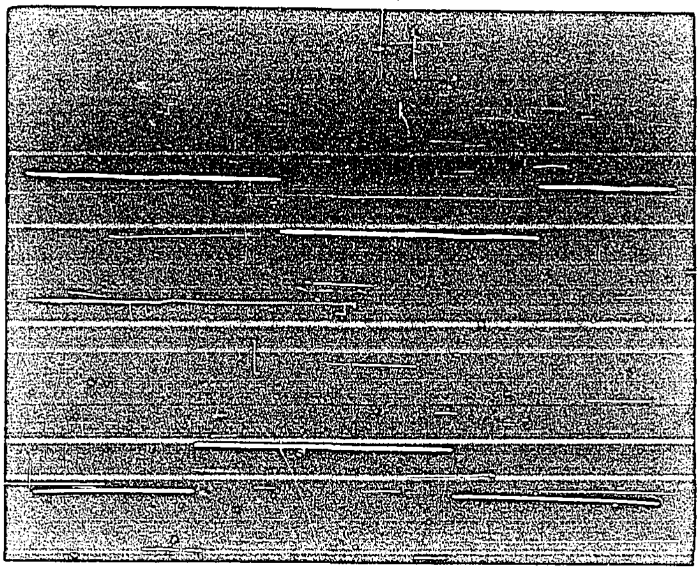


Fig. 18. 0.5 ms/cm

Upper - 50 v/cm 400 cps supply to left side of phase shift amplifier Fig. 11

Note: A parallel L-C resonant network is used in series with the 400 cps source and a series L-C resonant network is connected across the a-c input to the phase-shift amplifier to filter the supply to the amplifier.

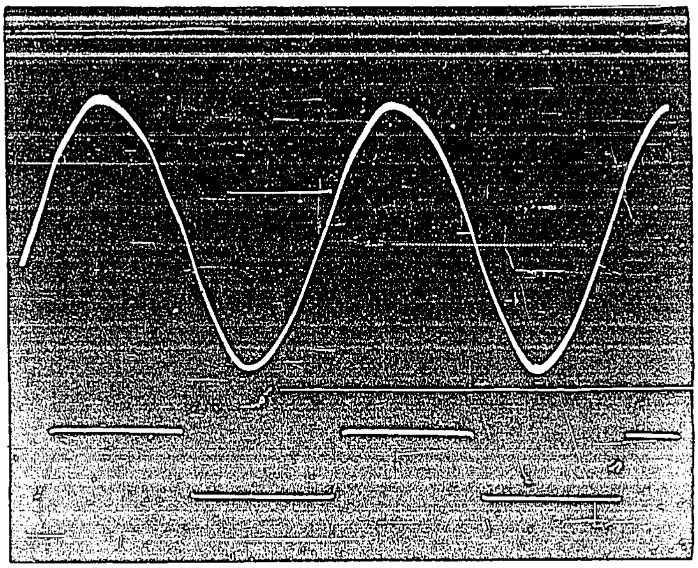
Lower - 10 v/cm voltage across 1N1771 Zener on left side of phase-shift amplifier

Fig. 19. 0.5 ms/cm, 10 v/cm both traces

Upper - output voltage across left-hand 220 ohm resistor of flip-flop in phase-shift amplifier

Lower - voltage across left-hand 0.047 μf capacitor in phase-shift amplifier

Note: Master oscillator frequency control now set at minimum



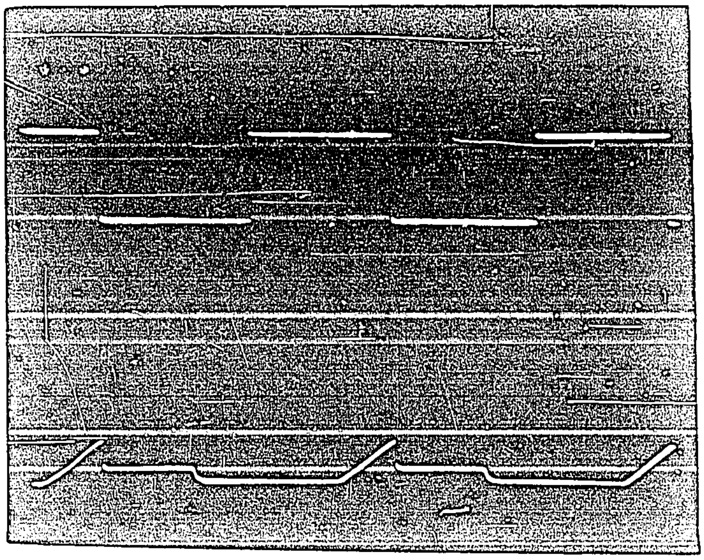


Fig. 20. 20 ms/cm, 10 v/cm

voltage across one half of secondary of output transformer in high frequency inverter Fig. 13

Fig. 21. 0.5 ms/cm

Upper - 100 v/cm anode to cathode voltage of SCR1

Lower - 2 v/cm gate to cathode voltage of SCR1

Note: Frequency control set at minimum

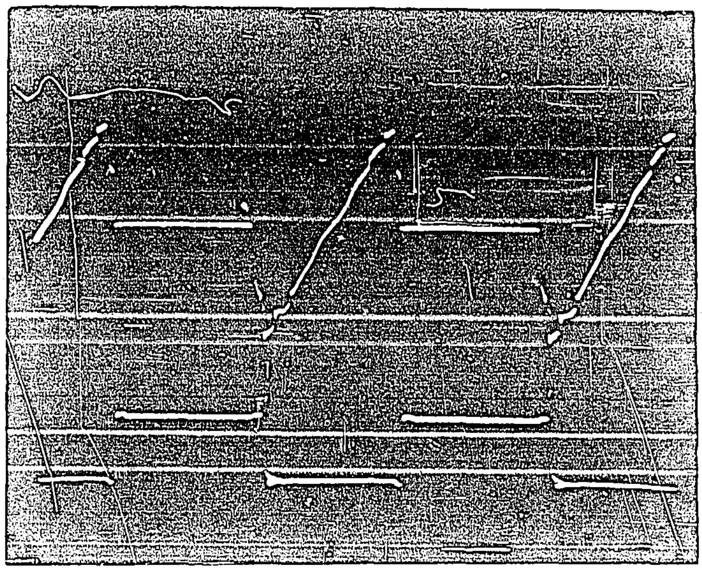


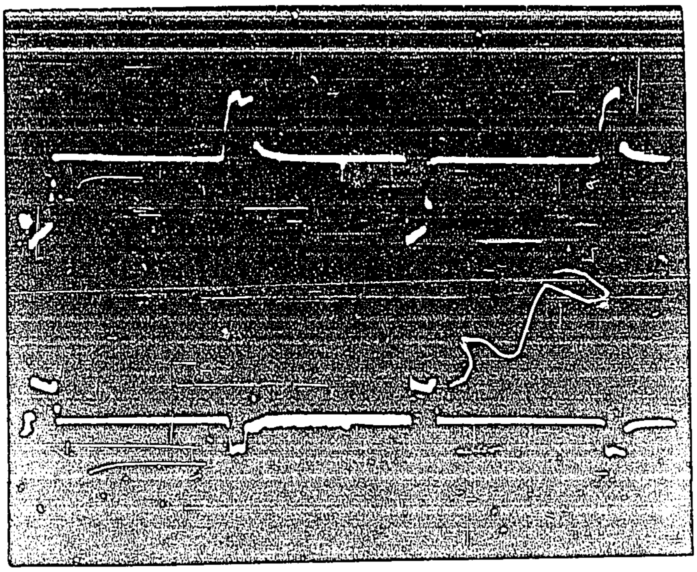
Fig. 22. 0.2 ms/cm, 50 v/cm both traces

Upper - white motor line to neutral voltage

Lower - orange motor line to neutral

Note: frequency control set at minimum and relatively low motor voltage

Fig. 23. Same as Fig. 22 except higher motor voltage



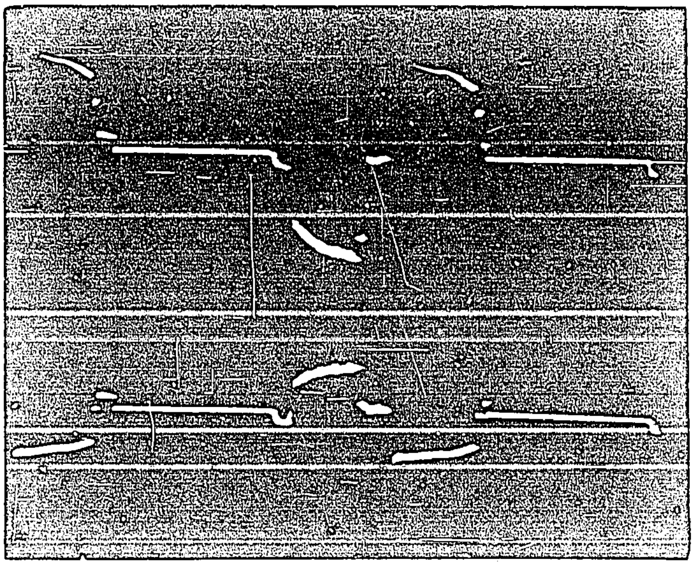


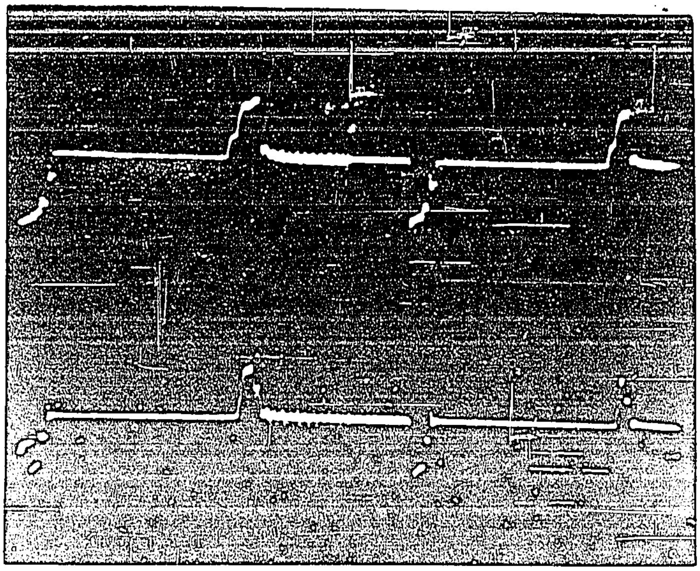
Fig. 24. 0.2 ms/cm, 100 v/cm both traces

Upper - voltage from white to blue motor line

Lower - voltage from orange to blue motor line

Note: frequency control set at minimum and relatively low motor voltage

Fig. 25. Same as Fig. 24 except higher motor voltage



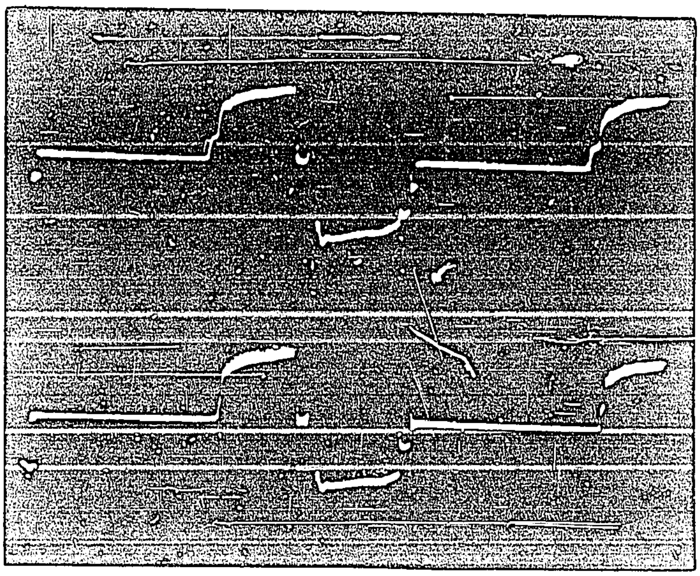


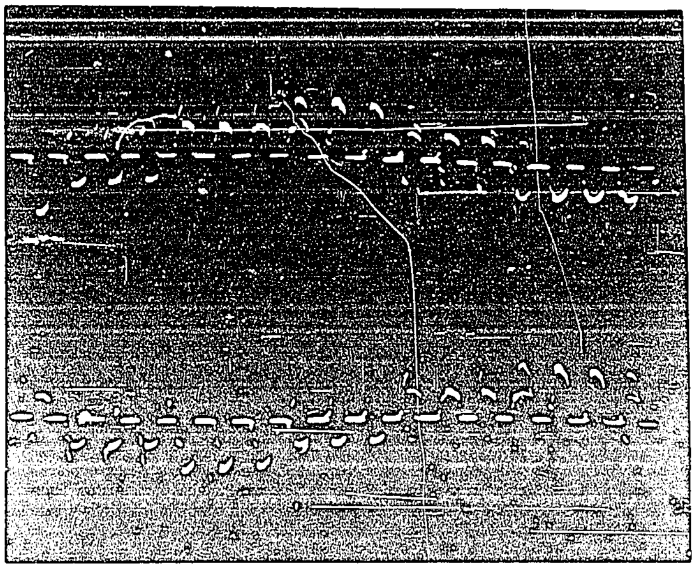
Fig. 26. 2 ms/cm, 100 v/cm both traces

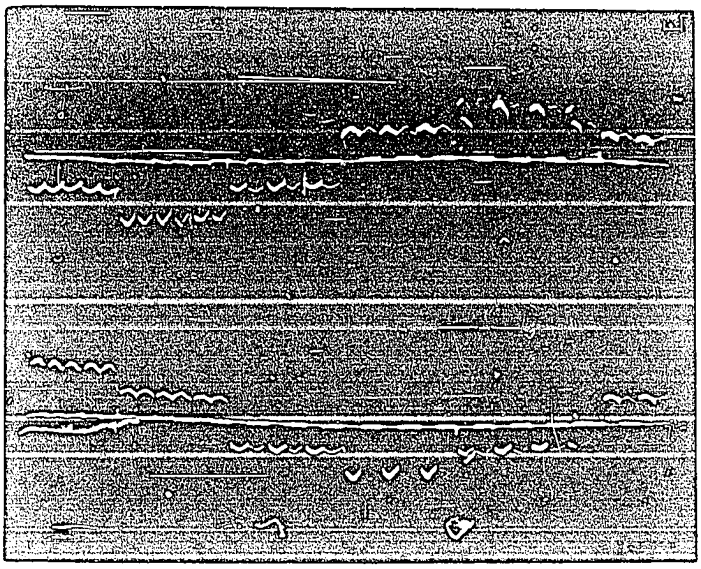
Upper - white motor line to neutral voltage

Lower - orange motor line to neutral voltage

Note: Adjusted motor speed to 1359 RPM so could synchronize scope, and motor voltage relatively low

Fig. 27. Same as Fig. 26 except higher motor voltage





occurred were observed and the values of speed and fundamental motor voltage were recorded for several different operating conditions. The output from the SCR power supply is limited to about 38 volts RMS line to neutral on the motor because of the 400 cps voltage level used in conjunction with the voltage drops present in the circuit and the commutating notches required in the voltage wave. Table 12 shows the test results.

Table 12. SCR power supply test results

RPM	Load rack ohms in series with each line	RMS fundamental component of motor line to neutral volts	Operating condition
730	0	15	no hunting
730	0	18	no hunting
730	0	23	no hunting
730	0	25	slight hunting
1820	0	26	hunting
1820	0	32	no hunting
1820	0	38	no hunting
730	1.5	16	no hunting
730	1.5	21	no hunting
730	1.5	26	hunting
730	1.5	30	no hunting
1800	1.5	25	hunting
1800	1.5	37	slight hunting
740	6	24	slight hunting
740	6	30	no hunting
1800	6	28	hunting
1800	6	38	slight hunting

The hunting is observed both by noting an oscillation of the ammeters in series with the motor line, and with a strobotac directed at the rotor through the end shield of the motor. As observed by comparing Table 12 with Table 11, the hunting occurs at approximately the same operating conditions whether the motor is being supplied from the M-G set or the SCR supply. The slight differences which do occur are caused most probably either by the effects of the harmonics in the output of the SCR supply or by the regulation of the 400 cps supply. Appreciable harmonics are present in the output of the SCR supply as indicated by the following typical values of the larger amplitude harmonics:

Fundamental--38 volts RMS @ 60 cps
--6 volts RMS @ 300 cps
--25 volts RMS @ 780 cps
--20 volts RMS @ 900 cps

These were measured using the Hewlett Packard Wave Analyzer with the motor running at the seventh operating condition listed in Table 12.

V. CONCLUSIONS

Although it was previously suspected and partially verified by test, this research has confirmed that the hunting of a synchronous motor operated from a variable frequency supply is an inherent characteristic of the machine. The hunting may occur whether an M-G set, an SCR supply, or any similar 3-phase supply is used. The range of operating conditions where hunting occurs does not appear to be markedly effected by the harmonics which are present in the output of an SCR supply.

In the analytical prediction of where hunting will occur, the major difficulty is knowing the precise values of the motor constants. From 40% to 100% speed, there is good correlation between the computer calculations and experimental results with an M-G set power supply, regarding the trends in the stability performance with changes in the system parameters. The following factors are believed to be most responsible for the differences between the precise operating conditions where hunting occurs with the motor connected to the M-G set supply as compared with that predicted from the computer calculations:

1. A satisfactory way of measuring accurately certain of the constants, particularly the stator-rotor mutual inductance, has not been found. However, this parameter can be varied over a wide range without

materially changing the points where hunting occurs.

2. Saturation of the machine is believed to cause a reduction in the machine inductances, with the greatest effect on L_d, as the applied voltage is increased. When the machine inductances change in this fashion, there is a significant effect on the stability performance as predicted by the analysis.

3. The theorem used in the analysis is a sufficient but not necessary condition for stability. Thus, the analysis may not predict asymptotic stability as often as it actually occurs. However, this is a very practical engineering result as the analysis will give conservative results in general.

It is believed that excellent correlation could be achieved between the analytical and experimental results with a specially constructed reluctance motor with no windings on the rotor, negligible saturation, and specially selected iron to minimize core losses.

In this research, one of the simpler theorems due to Liapunov is used as the heart of the stability analysis. Although, the computations are involved, it has provided a rigorous and straight-forward method for analyzing a very non linear control system--the reluctance synchronous motor. This has proved to be an excellent application of Liapunov's stability theory since it is possible to handle this highly non linear problem with reasonably good correlation between predicted and experimental results. There is much additional interesting work indicated as a result of this research including particularly the determination of suitable Liapunov functions to extend the stability analysis, to study the transient behavior of the motor, and in optimization studies. It should also be possible to apply Liapunov's stability theory more broadly to a number of other inverter and cycloconverter a-c motor controls as well as to general studies of the transient behavior of salient pole synchronous machines, as both of these situations involve reasonably complicated systems of non linear differential equations.

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